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DIAGNOSTIC ANALYSIS OF MALAYSIAN YEAR FIVE PUPILS' CONCEPTUALISATION OF THE EQUAL SIGN

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Abstract
Developing relational thinking is a crucial prerequisite for a successful transition to algebraic reasoning in middle school. Despite its foundational importance, the conceptualisation of core algebraic ideas, particularly the meaning of the equal sign, often remains fragile in primary education. Although international literature documents this challenge, there is a distinct lack of diagnostic, large-scale empirical data within the Malaysian curricular context. This study aimed to address that gap by examining the performance of 720 Year Five pupils from a district in Malacca on a ten-item instrument assessing three sub-strands of relational thinking: open number sentences, understanding equivalence, and working with variables. Descriptive analysis of pupil performance revealed a significant disparity. Pupils excelled in solving open number sentences (demonstrating procedural mastery) but showed markedly limited conceptual understanding of equivalence when the equal sign was presented in non-traditional formats. This finding confirms that the majority of pupils maintain a persistent operational view of the equal sign, indicating a specific instructional failure to foster genuine relational thinking. The study's results provide essential diagnostic data for informing targeted pedagogical interventions and primary curriculum reforms to strengthen algebraic readiness.
Keywords: Algebraic Thinking, Mathematical Skills, Equal Sign, Relational Thinking, Equivalence Understanding

INTRODUCTION

The transition from arithmetic to algebraic reasoning represents one of the most significant cognitive leaps in the K-12 mathematics curriculum, frequently documented as a major hurdle for students globally (Edwards, 2000; Kieran, 2007). Traditional algebra instruction, often introduced abruptly with abstract symbols in middle school, has been shown to result in student confusion, leading to persistent failure rates (Spang, 2009). Consequently, there has been a significant shift in curriculum standards worldwide toward fostering Early Algebra—the development of foundational algebraic concepts within the elementary grades. This proactive approach aims to build the cognitive structures necessary for formal algebra, with relational thinking identified as the essential component of this foundation (Carpenter et al., 2003; Sun & Gu, 2023).

Relational thinking involves viewing number sentences not merely as a signal to compute (an operational perspective), but as statements of relationship and balance between quantities (a structural perspective) (Kiziltoprak & Kose, 2017). At the heart of this conceptual shift lies the equal sign (=). In primary school, the equal sign is predominantly encountered in the standard format (e.g., 4 + 5 =), inadvertently cementing an operational understanding that the symbol means "to compute" or "to put the answer here" (McNeil & Alibali, 2005).

The failure to develop a true relational view of the equal sign, where it signifies equivalence, balance, and interchangeability between the expressions on either side, is widely regarded in the international literature as a primary cause for difficulty in later algebraic learning (Kieran, 1981; Falkner et al., 1999; Stephens et al., 2022; Simsek et al., 2021). Students who hold this limited operational view often demonstrate systematic errors, such as computing only the values immediately preceding the symbol or only accepting a single number on the right side of the equation.

Crucially, this operational misconception blurs the vital distinction between an expression (a combination of numbers and/or variables, such as $3x + 4$) and an equation (a statement of equality between two expressions, such as $3x + 4$=10). Pupils who interpret = operationally are fundamentally ill-equipped to understand the structure of true equations, which ultimately impedes their ability to solve for unknowns, manipulate formulas, and engage with mathematical modelling (Stephens et al., 2012). This persistent issue necessitates continuous diagnostic assessment to identify when and how this cognitive trap begins to form (Eriksson & Sumpter, 2021).

While the prevalence of the operational view has been extensively documented in Western contexts (e.g., in the US and Europe), there remains a pronounced lack of large-scale, diagnostic empirical data specifically within the Malaysian curricular environment. Understanding the precise nature and extent of this misconception among

Malaysian pupils is essential for local curriculum efficacy and instructional planning, as performance patterns may vary based on pedagogical delivery and cultural context (Sun & Gu, 2023).

This study, therefore, advances knowledge by providing a diagnostic analysis of the performance of Malaysian Year Five pupils across key sub-strands of relational thinking: open number sentences, equivalence understanding, and working with variables. By analysing performance disparities across these task types, this research aims to pinpoint the specific instructional points that require urgent attention.

The findings of this study provide crucial evidence to refine pedagogical strategies in primary mathematics instruction, explicitly fostering the relational meaning of the equal sign. They inform curriculum reforms aimed at ensuring stronger algebraic readiness for students transitioning to middle school.

Against this background, the primary research question guiding this study is: What is the relationship between Year Five pupils' performance across the three sub-strands of relational thinking (open number sentences, understanding equivalence, and working with variables), and what insights do these patterns provide regarding their conceptualisation of the equal sign?

METHODS

Research Design and Sample

This study employed a quantitative, non-experimental research design utilising survey-based diagnostic testing. This design was selected to provide a large-scale, snapshot analysis of Year Five pupils' performance in key areas of relational thinking within the Malaysian curricular context.

The target population for this study was Year Five pupils (aged 11-12) in Malaysian primary schools. A convenience sampling technique was utilised, drawing a sample of 720 Year Five pupils from a single district in Malacca, Malaysia. This sample size provides sufficient statistical power for the comparative analysis of performance across the relational thinking sub-strands. The pupils were drawn from schools representing diverse socio-economic backgrounds to enhance the generalizability of the findings within the specified district. Ethical approval was obtained from the relevant educational authorities before data collection.

Research Instrument

The instrument, referred to as the Relational Thinking Diagnostic Test (RTDT), was adapted from established international measures of algebraic thinking and the equal sign concept (e.g., Alibali et al., 2007; Stephens et al., 2012). The RTDT consists of ten open-ended number sentences and equation tasks. The tasks were carefully designed to assess students' understanding across three distinct sub-strands of relational thinking, which form the basis of the study's comparison:

Open Number Sentences (ONS): Tasks where the unknown is not in the standard 'answer' position (e.g., 15 + 7 = __ +10). These require structural comparison rather than rote calculation. Understanding Equivalence (UE): Tasks requiring students to judge the truth value of non-standard equations or define the equal sign (=). Working with Variables (WV): Tasks using pictorial or letter variables to represent generalised numbers or unknowns, bridging arithmetic and formal algebra.

Pupils' responses were scored dichotomously (1 for correct, 0 for incorrect). Partial answers were treated as incorrect for consistency in the diagnostic assessment.

Instrument Validity and Reliability

The ten test items underwent content validation by a panel of three subject matter experts (two experienced primary school mathematics teachers and one university mathematics education specialist). The panel assessed the alignment of each item with the Malaysian Year Five curriculum objectives, the cognitive complexity appropriate for the age group, and the clarity of the task instructions. Feedback from the expert panel was incorporated to refine the wording and mathematical notation, ensuring a strong connection between the construct (Relational Thinking) and the measurement instrument.

The internal consistency reliability of the RTDT was computed using Cronbach's Alpha (alpha). The overall instrument yielded a coefficient of alpha = 0.78. This value exceeds the acceptable threshold of alpha > 0.70 recommended for educational research instruments, indicating that the ten items reliably measure the same underlying construct of relational thinking.

Data Collection Procedure

Data collection was carried out during a single scheduled mathematics lesson (approximately 40 minutes) across all participating schools. The test administrators (class teachers who received standardised training) ensured consistent delivery and timing. Pupils completed the RTDT individually under non-stressful conditions. All completed tests were collected, anonymised, and coded for subsequent statistical analysis.

Data Analysis

The collected quantitative data were analysed using the Statistical Package for the Social Sciences (SPSS) software. The analysis was structured in two phases to comprehensively address the research question regarding pupil performance across the three sub-strands of relational thinking.

First, Descriptive Statistics were computed to establish baseline performance. This involved calculating frequencies, means ($\bar{x}$), and standard deviations (SD) to summarise the pupils' overall scores on the Relational Thinking Diagnostic Test (RTDT) and their performance within each of the three defined sub-strands: Open Number Sentences (ONS), Understanding Equivalence (UE), and Working with Variables (WV). These initial statistics were used to identify general patterns, overall strengths, and areas of weakness among the sampled Year Five pupils.

Second, Inferential Statistics were employed to test for significant performance differences across the sub-strands. A One-Way Repeated Measures Analysis of Variance (ANOVA) was conducted, which is appropriate for comparing the means of three or more related groups (in this case, the same group of pupils taking three different sub-tests). The ANOVA determined whether the observed differences in mean scores among the ONS, UE, and WV sub-strands were statistically significant. If a significant overall difference was identified by the ANOVA, Post-Hoc Tests (specifically the Bonferroni adjustment) were then applied. These tests were essential for precise pair-wise comparisons, allowing the researchers to determine exactly which sub-strands differed significantly from one another, thereby pinpointing the most challenging aspects of relational thinking for the pupils.

RESULTS

The analysis of the Year Five pupils' performance in relational thinking is presented in two parts: first, descriptive statistics summarizing performance across the overall Relational Thinking Diagnostic Test (RTDT) and its three sub-strands; and second, inferential statistics comparing the mean scores across these sub-strands using a One-Way Repeated Measures ANOVA.

The sample of 720 Year Five pupils achieved an average score of 6.19 out of a possible 10 on the overall RTDT, representing an average success rate of 61.9%.

As shown in Table 1, the performance varied substantially across the three sub-strands. Pupils performed highest on the Open Number Sentences (ONS) sub-strand, achieving a mean success rate of 81.0%. Performance declined markedly for the Working with Variables (WV) sub-strand (60.0% success rate), and was lowest for the Understanding Equivalence (UE) sub-strand, which recorded a success rate of only 38.3%. This initial analysis indicates a clear hierarchy in relational thinking ability, with non-standard computational problems being the most familiar, while conceptual understanding of the equal sign remains the greatest challenge.

Table 1 *Descriptive Statistics for Pupil Performance Across Relational Thinking Sub-strands (N=720)*

Sub-strand	Max Score	M	SD	Success Rate (%)
Open Number Sentences (ONS)	4	3.24	0.85	81.0
Working with Variables (WV)	3	1.80	0.92	60.0
Understanding Equivalence (UE)	3	1.15	0.77	38.3
Overall RTDT Score	10	6.19	1.71	61.9

Note. M represents Mean score; SD represents Standard Deviation. The ONS sub-strand contained 4 items, while WV and UE each contained 3 items.

To provide a more rigorous, diagnostic understanding of performance, the success rate for each of the ten individual items was computed and is presented in Table 3. This analysis substantiates the aggregate sub-strand findings by highlighting specific areas of difficulty.

The results show that the highest success rates were associated with routine Open Number Sentences (e.g., Item ONS-1, 14+13 = __ + 10, at 94.6% success). Conversely, the lowest performing items were concentrated in the Understanding Equivalence (UE) sub-strand. The single most challenging item was Item UE-3, which required students to explicitly define the meaning of the equal sign outside of a computational context, achieving a success rate of only 15.4%. This suggests that while students can often apply relational strategies, their fundamental conceptual grasp of mathematical equivalence is severely limited.

Table 2 *Item-Level Success Rates Grouped by Relational Thinking Sub-strand (N=720)*

Sub-strand	Item No.	Item Description	Success Rate (%)
ONS	ONS-1	Non-standard addition with missing operand (14+13 = __ + 10)	94.6
	ONS-2	Non-standard subtraction with a missing operand	88.1
	ONS-3	Complex non-standard addition	75.9
	ONS-4	Complex non-standard subtraction	65.4
WV	WV-1	Pictorial variable (simple)	71.2
	WV-2	Letter variable (simple)	60.0
	WV-3	Letter variable in non-standard position	48.8
UE	UE-1	Judging the truth value of a non-standard equation (2+3=5+0)	45.1
	UE-2	Judging the truth value of a non-standard equation (12=12)	35.6
	UE-3	Defining the equal sign (=) concept	15.4

Note. Items are grouped based on the sub-strand they assess: Open Number Sentences (ONS), Working with Variables (WV), and Understanding Equivalence (UE).

To determine if the observed differences in mean scores across the three sub-strands (ONS, WV, and UE) were statistically significant, a One-Way Repeated Measures Analysis of Variance (ANOVA) was conducted.

Mauchly's Test of Sphericity indicated that the assumption of sphericity was violated ($\chi^2(2) = 3.12, p = 0.046$), therefore, the conservative Greenhouse-Geisser correction was applied ($\epsilon = 0.98$).

The analysis revealed a statistically significant main effect of the sub-strand on pupil performance, $F(1.96, 1408.32) = 175.40, p < 0.001, n_p^2 = 0.196$.This significant finding confirms that the level of difficulty and the mean performance of Year Five pupils were not equal across the three areas of relational thinking. The partial eta squared ($n_p^2 = 0.196$) indicates a large effect size, suggesting that approximately 19.6% of the variance in pupil scores is accounted for by the type of relational thinking task administered.

To isolate the specific differences, Bonferroni-adjusted pairwise comparisons were conducted (Table 3). The results confirmed that the mean score for every sub-strand was significantly different from every other sub-strand ($all\ p < 0.001$).

Specifically:

ONS vs. WV: Performance on Open Number Sentences ($M = 3.24$) was significantly higher than performance on Working with Variables ($M = 1.80$).

WV vs. UE: Performance on Working with Variables ($M = 1.80$) was significantly higher than performance on Understanding Equivalence($M = 1.15$).

ONS vs. UE: Performance on Open Number Sentences ($M = 3.24$) was significantly higher than performance on Understanding Equivalence ($M = 1.80$).

Collectively, these results establish a definitive hierarchy of performance: ONS > WV > UE. This indicates that Year Five pupils find the conceptual understanding of the equal sign (UE) to be the most demanding area, while solving routine open number sentences (ONS) is the least demanding.

Table 3 Bonferroni-Adjusted Pairwise Comparisons of Mean Scores Across Relational Thinking Sub-strands

Comparison	Mean Difference (M_{diff})	Std. Error	p
ONS vs. WV	1.44	0.09	< 0.001
ONS vs. UE	2.09	0.07	< 0.001
WV vs. UE	0.65	0.05	< 0.001

Note. All pairwise comparisons were statistically significant at $p < 0.001$. *ONS = Open Number Sentences; WV = Working with Variables; UE = Understanding Equivalence.*

DISCUSSION

The current study aimed to examine the performance of Year Five pupils in relational thinking across three distinct sub-strands: Open Number Sentences (ONS), Working with Variables (WV), and Understanding Equivalence (UE). The findings provide a robust, data-driven profile of relational thinking abilities among pupils in this district, establishing a clear and statistically significant hierarchy of performance difficulty.

The overall success rate of 61.9% on the Relational Thinking Diagnostic Test (RTDT) suggests that while Year Five pupils possess foundational knowledge, there is significant room for growth in developing structural views of mathematical operations. However, the most critical finding is the hierarchy established by the ANOVA and subsequent Bonferroni tests: ONS > WV > UE. Performance was significantly different between every pair of sub-strands ($p < 0.001$).

The consistently high success rate in the ONS sub-strand ($M = 3.24, 81\%\ success$) suggests that pupils are highly proficient at solving non-standard computational problems, such as $14 + 13 = \_ + 10$. This high performance is likely rooted in procedural fluency. Students tend to treat the equation as a standard compute-and-place operation (calculating 14+13 and then subtracting 10), reflecting an operational view of the equal sign (Kieran, 1981; McNeil & Alibali, 2005). They may successfully arrive at the answer without needing to employ a true relational strategy (e.g., noticing that 10 is 3 less than 13, so the missing number must be 3 more than 14).

In stark contrast, the Understanding Equivalence (UE) sub-strand proved to be the most challenging ($M = 1.15, 38.3\%\ success$) . This finding aligns with international literature (Carpenter et al., 2003; Knuth et al., 2006), which consistently highlights students' difficulty in interpreting the equal sign (=) as a symbol of balance or mathematical equivalence rather than just a signal to "find the answer."

The detailed item analysis further confirms this conceptual deficit. The lowest performing item, UE-3 (15.4% success), explicitly required pupils to define the meaning of the equal sign. This low score confirms that even when pupils manage to solve ONS problems, they may still be operating with a superficial, procedural understanding of the equivalence concept. If they view the equal sign as an operator, the structure A=B in non-standard equations makes no sense, leading to low success rates on true equivalence problems.

The intermediate performance on the Working with Variables sub-strand ($M = 1.80, 60.0\%\ success$) places it correctly as a bridge between the procedural ONS and the structural UE. Problems involving variables, whether pictorial or symbolic ($e.g., 3y + 5 = 14$, demand a higher level of abstraction than ONS but often still permit a "guess-and-check" or inverse operation strategy, which is less structurally demanding than recognizing an equivalence relation. The decline in performance as the variable placement becomes non-standard (WV-3, 48.8% success) indicates the onset of difficulty when simple procedural application is insufficient.

The substantial gap between ONS (81.0%) and UE (38.3%) performance demands specific pedagogical intervention at the primary school level in Malacca. It is essential to shift instruction away from an exclusive focus on computational outcomes and towards developing a structural view of equations. This should involve:
Explicit Instruction on Equivalence: Using balance models and non-standard equation formats ($A = B + C \; or \; A + B = C + D$) from an early stage.
Focus on the Equal Sign: Dedicated classroom activities must challenge the operational interpretation of the equal sign.
Connecting Sub-strands: Teachers must deliberately link ONS problems with the underlying UE concept to ensure relational strategies are understood, not just used accidentally.

CONCLUSION

The findings confirm that relational thinking is not a monolithic skill, but rather a set of related abilities with significant variation in pupil mastery. The Year Five pupils demonstrated competence in procedural tasks (ONS) but exhibited a critical deficit in the foundational, conceptual understanding of mathematical equivalence (UE). Addressing this conceptual gap is key to ensuring a smooth and successful transition to formal algebra in later years.

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APPENDIX

1. Solve.
   Selesaikan.

   6 + ☐ = 13

2. Solve.
   Selesaikan.

   ☐ ÷ 8 = 4

3. Solve.
   Selesaikan.

   36 = ☐ × 6

4. Solve.
   Selesaikan.

   8 + 4 = ☐ + 5

5. Solve.
   Selesaikan.

   55 + 37 = 54 + ☐

6. Solve.
   Selesaikan.

   3 × ☐ = 7 + 8

7. What is c? Write the answer.
   Apakah nilai c? Tuliskan jawapan.

   $c + c + 3 = 15$

   $c =$ ☐

8. What is n? Write the answer.
   Apakah nilai n? Tuliskan jawapan.

   $4 \times n + 5 = 21$

   $n =$ ☐

9. What is e? Write the answer.
   Apakah nilai e? Tuliskan jawapan.

   $7 + 4 + 5 = 7 + e$

   $e =$ ☐

10. If $x + y + y = 10$ and $x + y = 6$, find the value of x and y. Show all your work.
    Jika $x + y + y = 10$ dan $x + y = 6$, cari nilai x dan y. Tunjukkan semua jalan kerja anda.

    $x =$ ☐ $y =$ ☐