

# GROUP ACCEPTANCE SAMPLING PLANS FOR LIFETIMES FOLLOWING EXPONENTIATED WEIBULL DISTRIBUTION

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## Abstract:

A group acceptance sampling plan was created for the Exponentiated Weibull distribution, focusing on truncated lifetimes with a predetermined shape parameter. We determined the minimum number of groups needed for a specified group size and acceptance number, considering a set consumer risk and test termination time. The operating characteristic function values were computed for different quality levels, and we found the minimum ratios of actual average life to specified life at a given producer's risk. Tables illustrate these findings.

**Key words:** Exponentiated Weibull distribution, truncated lifetimes, group acceptance sampling plan, operating characteristic function, producer's risk.

## 1. INTRODUCTION

Quality control involves inspecting products or services to ensure they meet standards. When inspecting every single item isn't practical, sampling becomes necessary. This raises questions about the optimal sample size or whether sampling is even needed. By analyzing sample data, decisions can be made about accepting or rejecting an entire batch. Statistical quality control relies heavily on acceptance sampling, a methodology popularized by Dodge and Romig during World War II, when the US military used it to test ammunition. This approach enables informed decisions about product lots based on sample inspections.

Acceptance sampling often involves truncated life tests to determine the required sample size for a given lot, ensuring reliable quality control decisions. Traditional sampling plans typically involve testing individual items. However, to optimize time and resources, testers that can handle multiple items simultaneously are employed. The number of items per tester is usually predetermined by specifications. When sampling plans are designed for such multi-item testers, they're referred to as group acceptance sampling plans (GASPs). When GASPs are combined with truncated life tests, the lifetime of the product is assumed to follow a specific probability distribution. In this context, determining the sample size is equivalent to determining the number of groups required, where the sample size  $n=rg$  is calculated as the product of the group size and the number of groups.

Group acceptance sampling plans (GASP) have received considerable attention in reliability testing due to their efficiency in simultaneously testing multiple items. Various lifetime distributions have been explored in the context of GASP, each providing different levels of flexibility and modeling capabilities. Aslam and Jun [4], [6], [7], [8] laid foundational work in the development of group acceptance sampling plans under truncated life tests by utilizing distributions like the inverse Rayleigh, log-logistic, Weibull, and generalized exponential. Their contributions emphasized the practical utility of GASP in quality control settings. Subsequent works extended these frameworks to more flexible lifetime distributions. For instance, Almarashi et al. [3] introduced GASP based on the Marshall-Olkin Kumaraswamy exponential distribution, highlighting its increased flexibility over traditional models. Likewise, Fayomi and Khan [13] developed GASP for the generalized transmuted lifetime distribution, showcasing improvements in modeling skewed data. Rao and colleagues [28], [29], [30], [31] contributed significantly by formulating GASP under distributions like Half Normal, Weibull, Gompertz, and via Bayesian approaches. These models addressed varying shapes of hazard functions and incorporated prior knowledge, enhancing the robustness of acceptance decisions. The Weibull distribution, known for its adaptability, has been widely used in GASP studies. Rao [28], and Kantam and Rosaiah [19] demonstrated its utility in truncated life tests. Meanwhile, Jun et al. [14], [15] developed variable sampling plans for Weibull-distributed lifetimes under sudden death testing, optimizing test efficiency when test positions are limited. More recently, Algarni [1] proposed a GASP based on a compounded three-parameter Weibull model, advancing the existing Weibull-based models by allowing for increased shape

flexibility. Similarly, Alghamdi et al. [2] introduced a more flexible Lomax distribution for GASP, enhancing reliability assessment under skewed lifetime data. Distributions such as the inverse log-logistic [22], finite mixture models [23], [41], and exponentiated forms [36], [42] have also been explored in GASP frameworks to better fit real-life reliability data. The use of exponentiated distributions, like the exponentiated inverted Weibull [42] and exponentiated log-logistic [36], closely aligns with the motivation for using the exponentiated Weibull distribution in this study. Additional studies have expanded GASP to include double sampling [22], [39], [40], two-stage plans [37], and hybrid strategies [35], enhancing decision-making efficiency. Monte Carlo simulations and sudden death testing strategies have also been used to assess the performance of sampling plans under restricted conditions [24], [25], [43], [45].

This study focuses on designing a group acceptance sampling plan (GASP) for truncated life tests, assuming that product lifetimes follow a Type II Exponentiated Weibull Distribution (EWD). The paper is organized as follows: Section 2 introduces the Type II EWD, while Section 3 outlines the proposed GASP. Section 4 illustrates the methodology using real-world data, and Section 5 summarizes the key findings and conclusions

## 2. Exponentiated Weibull Distribution

The Exponentiated Weibull Distribution (EWD) was indeed introduced by Mudholkar and Srivastava as an extension of the traditional Weibull distribution, adding more flexibility to model various types of data. The applications of this distribution in reliability and survival studies were illustrated by Mudholkar et al. The exponentiated exponential distribution and the Weibull distribution, examined by Gupta et al, are the special cases of EWD. Mudholkar and Hutson demonstrated that the EWD has bathtub shaped and unimodal failure rates. The probability density function (pdf) and cumulative distribution function (cdf) of EWD are defined as follows.

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$$f(t; \sigma, \lambda, \theta) = \lambda \theta (t)^{\lambda-1} e^{-(t)^\lambda} \left[ 1 - e^{-(t)^\lambda} \right]^{\theta-1}; \quad t > 0 \text{ and } \lambda, \theta > 0. \quad (1)$$

$$\text{and } F(t; \lambda, \theta) = \left[ 1 - e^{-(t)^\lambda} \right]^\theta; \quad t > 0 \text{ and } \theta > 0 \quad (2)$$

to discuss the three-parameter Exponentiated Weibull Distribution(EWD), Which includes a scale parameter  $\sigma$  in addition to two scale parameters denoted by  $\lambda$  (lambda) and  $\theta$  (theta). The modified three-parameter Exponentiated Weibull Distribution (EWD) allowing for greater flexibility in modeling real-world data, whose pdf and cdf are respectively given by

$$f(t; \sigma, \lambda, \theta) = \frac{\lambda \theta}{\sigma} \left( \frac{t}{\sigma} \right)^{\lambda-1} e^{-(t/\sigma)^\lambda} \left[ 1 - e^{-(t/\sigma)^\lambda} \right]^{\theta-1}; \quad t > 0 \text{ and } \sigma, \lambda, \theta > 0. \quad (3)$$

$$\text{and } F(t; \sigma, \lambda, \theta) = \left[ 1 - e^{-(t/\sigma)^\lambda} \right]^\theta; \quad t > 0 \text{ and } \theta > 0 \quad (4)$$

where  $\sigma$  (sigma) is a scale parameter, and  $\lambda$  (lambda) and  $\theta$  (theta) are the shape parameters. The exponentiated Weibull distribution is a special case of (1) for  $\lambda = 1$  and the Weibull distribution is a special case of (1) for  $\theta = 1$ . For  $0 < q < 1$ , the 100q-th percentile or q<sup>th</sup> quantile of the EWD, denoted by  $t_q$ , is given by

$$F(t_q) = q \Rightarrow t_q = F^{-1}(q) = \sigma \eta_q, \quad \text{where } \eta_q = \left[ -\ln(1 - q^{1/\theta}) \right]^{1/\lambda} \quad (5)$$

From this equation, it's evident that for fixed values of  $\lambda = \lambda_0$  and  $\theta = \theta_0$ , the quantile  $t_q$  is dependent on the scale parameter  $\sigma$  and by setting a specific value for the quantile  $t_q$  say  $t_q^0$ , we can solve for the corresponding value of the scale parameter  $\sigma$ , say  $\sigma_0$ , denoted as

$$\sigma_0 = t_q^0 / \left[ -\ln(1 - q^{1/\theta_0}) \right]^{1/\lambda_0}$$

Clearly,  $t_q \geq t_q^0 \Leftrightarrow \sigma \geq \sigma_0$  and we may also note that  $\sigma_0$  are influenced by  $\lambda_0$  and  $\theta_0$ . Notably, the relationship between these parameters and the variables they depend on is crucial in developing acceptance sampling plans for the Exponentiated Weibull Distribution.

A key consideration is ensuring that  $t_q$  exceeds  $t_q^0$  equivalently  $\sigma$  exceeds  $\sigma_0$  surpassing its corresponding threshold.

## 3. The Group Acceptance Sampling Plan (GASP)

This discussion focuses on group acceptance sampling plans (GASPs) for products with lifetimes following an Exponentiated Weibull Distribution (EWD). Specifically, we explore GASPs under truncated life testing, where the

decision criterion is based on the cumulative number of failures across all groups. The step-by-step procedure for implementing this plan is outlined below, drawing on the work of Aslam et al. (2011).

The proposed plan involves the following steps:

1. Randomly select a sample of  $n$  products and divide them into  $g$  groups, each containing  $r$  items ( $n = r \times g$ ).
2. Determine an acceptance number  $c$  and a predetermined test termination time ( $t_0$ ).
3. Accept the lot if the total number of failures across all groups does not exceed  $c$ .
4. If more than  $c$  failures occur before the test termination time ( $t_0$ ), terminate the experiment and reject the lot.

That's a crucial step in evaluating the effectiveness of a group acceptance sampling plan.

The lot acceptance probability calculation typically involves considering the number of failures across all groups within the truncated life test before the prefixed time ( $t_0$ ). To delve deeper into the calculation specifics or discuss the implications of this approach as follows

$$P(p) = \sum_{i=0}^c \binom{rg}{i} p^i (1-p)^{rg-i} \tag{6}$$

In this context, the key parameters are:  $g$  (number of groups),  $c$  (acceptance number),  $r$  (group size), and  $p$  (probability of failure within the test termination time). Assuming the product's lifetime 't' follows an Exponentiated Weibull Distribution (EWD) then  $p = F(t; \sigma, \lambda, \theta)$ . Usually, it would be convenient to determine the experiment termination time  $t_0$ , as  $t_0 = \delta_q^0 t_q^0$  for a constant  $\delta_q^0$  and the targeted 100q-th lifetime percentile,  $t_q^0$ . Let  $t_q$  be the true 100q-th lifetime percentile. Then,  $p$  can be rewritten as

$$p = 1 - \left[ \exp \left[ - \left( \frac{t_0}{\sigma} \right)^\lambda \right] \right]^\theta = 1 - \left[ \exp \left[ - \left( \frac{\eta_q \delta_q^0}{(t_q/t_q^0)} \right)^\lambda \right] \right]^\theta \tag{7}$$

To determine the design parameters of the proposed group acceptance sampling plan, a two-point approach on the operating characteristic (OC) curve is employed, taking into account both producer's and consumer's risks. This approach evaluates quality levels based on the ratio of percentile lifetime to its specified lifetime;  $t_q/t_q^0$ . From the producer's perspective, the probability of lot acceptance should be at least a certain threshold should be at  $1 - \alpha$  when the product meets the acceptable reliability level (ARL), ensuring a high likelihood of acceptance for products that meet quality standards at  $p_1$ . The producer requires a high acceptance probability for lots meeting certain reliability standards, say  $t_q/t_q^0 = 2, 4, 6, 8$ , as outlined in equation (7). Conversely, the consumer prioritizes lot rejection probability should be at most  $\beta$  at the lot tolerance reliability level (LTRL),  $p_2$ . In this way, the consumer consider that a lot should be rejected when  $t_q/t_q^0 = 1$ , in equation (7). Now, let us consider

$$P_a(p_1) = \sum_{i=0}^c \binom{rg}{i} p_1^i (1-p_1)^{rg-i} \geq 1 - \alpha \tag{8}$$

$$P_a(p_2) = \sum_{i=0}^c \binom{rg}{i} p_2^i (1-p_2)^{rg-i} \leq \beta \tag{9}$$

Where  $p_1$  and  $p_2$  are given by

$$p_1 = \left[ 1 - \exp \left[ - \left( \frac{\eta_q \delta_q^0}{(t_q/t_q^0)} \right)^\lambda \right] \right]^\theta \text{ and } p_2 = \left[ 1 - \exp \left[ - (\eta_q \delta_q^0)^\lambda \right] \right]^\theta \tag{10}$$

The plan parametric quantities for distinct values of parameters are constructed. For a specified producer's risk  $\alpha = 0.05$  and termination time schedule  $t_0 = \delta_q^0 t_q^0$  with  $\delta_q^0 = 0.5$  and  $1.0$ , the parameter of the proposed group acceptance sampling plan are estimated for 50<sup>th</sup> percentiles at different confidence levels  $\beta = 0.25, 0.10, 0.05, 0.01$ . The plan parameters are presented in Tables 1, 2 and 3 for  $\lambda=1.5$  and  $\theta=1.5$ ,  $\lambda=2.5$  and  $\theta=1.5$  and  $\lambda=2.5$  and  $\theta=2.5$  at 50<sup>th</sup> percentile. Table 4 is constructed using ML estimates  $\hat{\lambda} = 0.689438941329603$ ,  $\hat{\theta} = 1.77779793377646$  at 50<sup>th</sup> percentile. We noticed from Table 1 to 4, we observe that the percentile ratio increases when the number of groups  $g$  reduces. Similarly, as  $r$  increases from 5 to 10, the number of groups reduces.

#### 4. Description of the Proposed Methodology with Real Lifetime Data

A dataset of remission times (in months) for 128 bladder cancer patients, originally presented by E.T. Lee (2003) [21], is analyzed below:

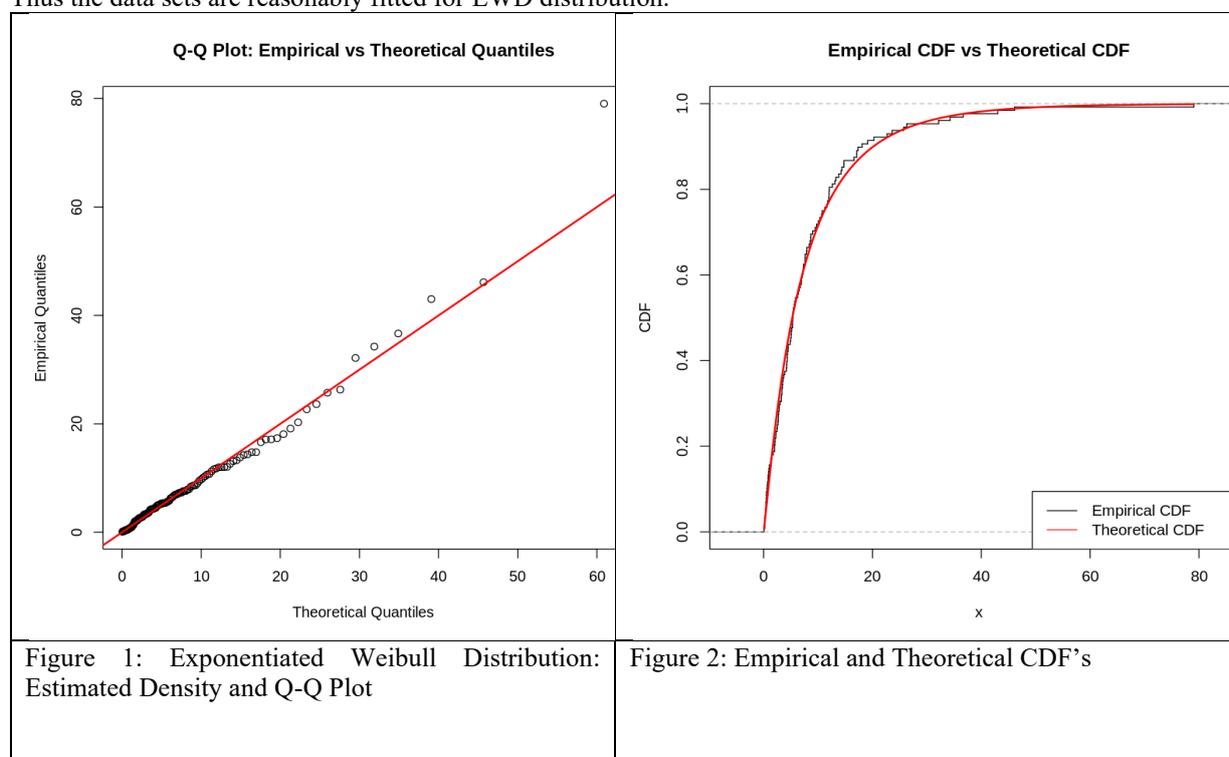
0.08, 2.09, 3.48, 4.87, 6.94, 8.66, 13.11, 23.63, 0.2, 2.23, 0.52, 4.98, 6.97, 9.02, 13.29, 0.4, 2.26, 3.57, 5.06, 7.09, 0.22, 13.8, 25.74, 0.5, 2.46, 3.46, 5.09, 7.26, 9.47, 14.24, 0.82, 0.51, 2.54, 3.7, 5.17, 7.28, 9.74, 14.76, 26.31, 0.81, 0.62, 3.28, 5.32, 7.32, 10.06, 14.77, 32.15, 2.64, 3.88, 5.32, 0.39, 10.34, 14.38, 34.26, 0.9, 2.69, 4.18, 5.34, 7.59, 10.66, 0.96, 36.66, 1.05, 2.69, 4.23, 5.41, 7.62, 10.75, 16.62, 43.01, 0.19, 2.75, 4.26, 5.41, 7.63, 17.12, 46.12, 1.26, 2.83, 4.33, 0.66, 11.25, 17.14, 79.05, 1.35, 2.87, 5.62, 7.87, 11.64, 17.36, 0.4, 3.02, 4.34, 5.71, 7.93, 11.79, 18.1, 1.46, 4.4, 5.85, 0.26, 11.98, 19.13, 1.76, 3.25, 4.5, 6.25, 8.37, 12.02, 2.02, 0.31, 4.51, 6.54, 8.53, 12.03, 20.28, 2.02, 3.36, 6.76, 12.07, 0.73, 2.07, 3.36, 6.39, 8.65, 12.63, 22.69, 5.49.

The above data set is assumed to follow EWD( $\lambda, \theta$ ). We obtain the MLEs of the parameters as

$\hat{\lambda} = 0.6894, \hat{\theta} = 1.777$ . To validate the model's fit, we conducted a Kolmogorov-Smirnov test on the dataset.

The results showed a Kolmogorov-Smirnov statistic of 0.0514 with a p-value of 0.8883, indicating that the Exponentiated Weibull Distribution (EWD) provides a reasonable fit to the data

Thus the data sets are reasonably fitted for EWD distribution.



Consider a product with a lifetime following an Exponentiated Weibull Distribution (EWD) with known index parameters  $\hat{\lambda} = 0.6894, \hat{\theta} = 1.777$ . The goal is to design a group acceptance sampling plan that ensures the 50th percentile of the lifetime distribution exceeds the remission time of bladder cancer patients. The producer's risk is 5%. For  $\hat{\lambda} = 0.6894, \hat{\theta} = 1.777$ , and the consumer's risk is 25%, with  $r=5, \delta_q^0 = 1.0$  and  $t_q / t_q^0 = 4$ , the minimum number of groups acceptance number given by  $g=5$  and  $c=4$  from Table 4, the plan's implementation involves selecting 20 products, divided into two groups of 10 items each. We can accept the lot when no more than 5 failures occur before 1 month from each of two groups. Based on this plan, the remission times have been accepted since there were only 5 failures before the 1-month termination time.

#### 5. CONCLUSIONS

This study introduces a group acceptance sampling plan for products with lifetimes following an Exponentiated Weibull Distribution (EWD). The design parameters,  $c$  and  $g$ , are determined using a two-point approach. Our results show that as the percentile ratio increases, the required number of groups ( $g$ ) decreases. Additionally, for certain parametric combinations, increasing the group size ( $r$ ) also leads to a reduction in the number of groups. The effectiveness of the proposed plan is demonstrated using real-world lifetime data, highlighting its practical applicability in industrial settings.

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**Table-1:** GASP for EWD  $\lambda=1.5$ ,  $\theta=1.5$  for 50<sup>th</sup> percentile.

$\beta$	$t_q/t_q^0$	r=5						r=10						
		$\delta_q^0 = 0.5$			$\delta_q^0 = 1.0$			$\delta_q^0 = 0.5$			$\delta_q^0 = 1.0$			
		c	G	pa	c	g	pa	c	g	pa	C	G	pa	
0.25	2	4	171	0.97307	-	0	0.00000	5	10	2	0.96973	7	4	0.964913
	4	2	5	0.98449	2	1	0.96009	2	1	0	0.97132	4	1	0.981468
	6	1	2	0.97062	2	1	0.99056	2	1	2	0.99376	3	1	0.985579
	8	1	2	0.98661	1	1	0.95714	1	1	4	0.97233	2	1	0.971320
0.10	2	4	283	0.95583	-	0	0.00000	5	16	3	0.95201	8	18	0.979846
	4	2	7	0.97836	3	4	0.98430	3	3	3	0.98913	4	1	0.981468
	6	1	3	0.95625	2	2	0.98121	2	2	2	0.98756	3	1	0.985579
	8	1	3	0.97998	1	1	0.95714	1	1	4	0.97233	2	1	0.971320
0.05	2	-	0	0.00000	-	0	0.00000	6	70	6	0.97552	8	23	0.974321
	4	2	9	0.97226	3	5	0.98041	3	4	6	0.98553	4	2	0.963280
	6	2	9	0.99457	2	2	0.98121	2	2	2	0.98756	3	1	0.985579
	8	1	4	0.97340	1	1	0.95714	2	2	0	0.99607	2	1	0.971320
0.01	2	-	0	0.00000	-	0	0.00000	6	107	2	0.96283	8	35	0.961186
	4	2	14	0.95718	3	7	0.97269	3	6	3	0.97838	4	2	0.963280

	6	2	14	0.99156 7	2	3	0.97195 7	2	3	0.98140 1	3	2	0.971366
	8	1	5	0.96686 3	2	3	0.99066 8	2	3	0.99411 1	2	1	0.971320

**Table-2:** GASP for EWD  $\lambda=2.5, \theta=1.5$  for 50<sup>th</sup> percentile.

M	$t_q/t_q^0$	r=5						r=10					
		$\delta_q^0 = 0.5$			$\delta_q^0 = 1.0$			$\delta_q^0 = 0.5$			$\delta_q^0 = 1.0$		
		c	G	pa	c	g	pa	c	g	pa	C	G	Pa
0.25	2	-	0	0.00000 0	-	0	0.00000 0	6	3	0.957403	9	5	0.9693 27
	4	2	2	0.95764 6	3	1	0.96274 0	3	1	0.961308	6	1	0.9855 95
	6	2	2	0.99054 4	3	1	0.99348 8	2	1	0.958237	4	1	0.9682 82
	8	1	1	0.97309 2	2	1	0.97859 4	2	1	0.985213	3	1	0.9613 08
0.10	2	-	0	0.00000 0	-	0	0.00000 0	7	13	0.970269	9	8	0.9513 77
	4	2	2	0.95764 6	4	4	0.98745 2	3	1	0.961308	6	1	0.9855 95
	6	2	2	0.99054 4	3	2	0.98701 9	2	1	0.958237	4	1	0.9682 82
	8	1	1	0.97309 2	2	1	0.97859 4	2	1	0.985213	3	1	0.9613 08
0.05	2	-	0	0.00000 0	-	0	0.00000 0	7	17	0.961300	-	0	0.0000 00
	4	3	8	0.98694 3	4	5	0.98434 0	4	2	0.986015	1	1	0.9855 95
	6	2	3	0.98584 9	3	2	0.98701 9	2	1	0.958237	1	1	0.9682 82
	8	2	3	0.99545 8	2	1	0.97859 4	2	1	0.985213	1	1	0.9613 08
0.01	2	0	0	0.00000 0	-	0	0.00000 0	8	95	0.978855	-	0	0.0000 00
	4	3	12	0.98047 8	4	7	0.97814 5	4	3	0.979095	6	2	0.9713 97
	6	2	4	0.98117 7	3	3	0.98059 2	3	2	0.987744	4	1	0.9682 82
	8	2	4	0.99394 8	2	2	0.95764 6	2	2	0.970644	3	1	0.9613 08

**Table-3:** GASP for EWD  $\lambda=2.5, \theta=2.5$  for 50<sup>th</sup> percentile.

$\beta$	$t_q/t_q^0$	r=5						r=10					
		$\delta_q^0 = 0.5$			$\delta_q^0 = 1.0$			$\delta_q^0 = 0.5$			$\delta_q^0 = 1.0$		
		c	g	pa	c	g	pa	c	g	pa	C	G	Pa
0.25	2	3	7	0.96332 2	-	0	0.00000 0	3	3	0.98616 3	8	2	0.975823
	4	1	1	0.97734 4	2	1	0.95076 9	1	1	0.98845 0	4	1	0.974406
	6	1	1	0.99579 2	2	1	0.99377 4	1	1	0.98233 9	3	1	0.991364
	8	1	1	0.99882 3	1	1	0.97734 4	1	1	0.99489 2	2	1	0.988450
0.10	2	4	68	0.98408 8	-	0	0.00000 0	5	5	0.97704 6	8	3	0.963954

	4	1	2	0.95520 1	2	1	0.95076 9	2	1	0.98845 0	4	1	0.974406
	6	1	2	0.99160 2	2	1	0.99377 4	1	1	0.98233 9	3	1	0.991364
	8	1	2	0.99764 7	1	1	0.97734 4	1	1	0.99489 2	2	1	0.988450
	2	4	88	0979456	-	0	0.00000 0	5	6	0.97251 8	8	4	0.952230
0.05	4	1	2	0.95520 1	2	1	0.95076 9	2	1	0.98845 0	4	1	0.974406
	6	1	2	0.99160 2	2	1	0.99377 4	1	1	0.98233 9	3	1	0.991364
	8	1	2	0.99764 7	1	1	0.97734 4	1	1	0.99489 2	2	1	0.988450
	2	4	135	0.96865 7	-	0	0.00000 0	5	10	0.95461 8	9	18	0.979733
0.01	4	2	7	0.99188 6	3	3	0.98411 3	2	2	0.97703 3	4	1	0.974406
	6	1	3	0.98742 9	2	2	0.98758 6	1	1	0.98233 9	3	1	0.991364
	8	1	3	0.99647 2	1	1	0.97734 4	1	1	0.99489 2	2	1	0.988450
	2	4	135	0.96865 7	-	0	0.00000 0	5	10	0.95461 8	9	18	0.979733

**Table-4:** GASP for EWD with  $\hat{\lambda} = 1.109$ ,  $\hat{\theta} = 0.8408$  for 50<sup>th</sup> percentile.

$\beta$	$t_q/t_q^0$	r=5						r=10					
		$\delta_q^0 = 0.5$			$\delta_q^0 = 1.0$			$\delta_q^0 = 0.5$			$\delta_q^0 = 1.0$		
		c	g	pa	c	G	pa	c	g	Pa	C	g	Pa
0.25	2	-	0	0.00000 0	-	0	0.00000 0	9	431	0.971357	-	0	0.000000
	4	4	24	0.97734 2	4	5	0.95961 2	5	2	0.96798	6	1	0.957005
	6	3	5	0.97200 6	4	5	0.98817 7	4	1	0.972662	5	1	0.953636
	8	3	5	0986432	3	2	0.96951 8	4	1	0.987636	5	1	0.980713
0.10	2	-	0	0.00000 0	-	0	0.00000 0	9	715	0.952933	-	0	0.000000
	4	4	40	0.96252 3	-	0	0.00000 0	6	7	0.976386	7	3	0.973102
	6	3	8	0.95558 8	4	8	0.98115 1	5	3	0.984951	5	1	0.953636
	8	3	8	0.97838 0	3	3	0.95462 7	4	2	0.975426	5	1	0.980713
0.05	2	-	0	0.00000 0	-	0	0.00000 0	-	0	0.000000	-	0	0.000000
	4	4	52	0.95155 6	-	0	0.00000 0	6	9	0.969742	7	4	0.964297
	6	4	52	0.98679 2	4	10	0.97649 4	5	4	0.979985	6	2	0.979483
	8	3	10	0.97304 9	3	3	0.95462 7	4	3	0.963366	5	2	0.961798
0.01	2	-	0	0.00000 0	-	0	0.00000 0	-	0	0.000000	-	0	0.000000
	4	-	0	0.00000 0	-	0	0.00000 0	6	14	0.953329	8	14	0.983952
	6	4	79	0.98000 2	4	15	0.96494 9	5	6	0.970128	6	3	0.969383

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	8	3	15	0.95984 7	4	15	0.98577 8	4	4	0.951455	5	2	0.961798
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