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MODIFIED LOGNORMAL DISTRIBUTIO FUNCTION FOR MODELLING SOIL-FREEZING CHARACTERISTIC CURVE

[1]BAKYTKUL SERIKBEK, [2]ALFRENDO SATYANAGA, [3]SONNY IRAWAN, [4]JONG KIM, [5]BAYRAMOV

[1]DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING, NAZARBAYEV UNIVERSITY, ASTANA, KAZAKHSTAN
[2]DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING, NAZARBAYEV UNIVERSITY, ASTANA, KAZAKHSTAN
[3]DEPARTMENT OF PETROLEUM ENGINEERING, NAZARBAYEV UNIVERSITY, ASTANA, KAZAKHSTAN
[4]DEPARTMENT OF CIVIL AND ENVIRONMENTAL ENGINEERING, NAZARBAYEV UNIVERSITY, ASTANA, KAZAKHSTAN
[5]DEPARTMENT OF PETROLEUM ENGINEERING, NAZARBAYEV UNIVERSITY, ASTANA, KAZAKHSTAN

EMAIL: [1]bakytkul.serikbek@nu.edu.kz, [2]alfrendo.satyanaga@nu.edu.kz, [3]sonny.irawan@nu.edu.kz, [4]jong.kim@nu.edu.kz, [5]emil.bayramov@nu.edu.kz

ORCHID ID : [1]0009-0005-6309-7510, [2]0000-0003-3849-2525, [3]0000-0002-7444-868X, [4]0000-0002-8796-3832, [5]0000-0002-3804-5373

Abstract: SWCC (Soil-water characteristic curve) and SFCC (Soil-freezing characteristic curve) are the main concepts describing the properties of unsaturated and frozen construction material, respectively. SWCC is the relationship between water content and matric suction of the soil and can be used for the understanding of unsaturated soil characteristics while SFCC demonstrates the relationship between the unfrozen moisture content of soil and sub-freezing temperature and can be used to predict the frozen soil behavior. The objective of this study is to develop new equation for modelling SFCC incorporating physical meaning of SWCC variables. SFCCs of natural soil from Turan Avenue near Nazarbayev University, Astana and engineered soil (sand-kaolin mixtures) are utilized to evaluate the proposed model. The laboratory experiments were conducted by measuring soil moisture and suction of the investigated soils under different temperatures. The determination of SFCC using the proposed model was compared with various mathematical equations. In addition, the effect of soil type was studied by utilizing mathematical SFCCs for a variety of soils, including Astana soil, sand-kaolin combinations, and SFCC envelopes based on the Unified Soil Classification System. Fine-grained soils exhibited a higher unfrozen water content, even at temperatures below -10 °C, due to their superior water absorption capabilities. The new SFCC model provided an important theoretical advancement by accurately describing SFCC fluctuations during freezing. Its examination utilizing experimental data from Astana soil revealed a better fit than the Fredlund & Xing (1994) SFCC model. A similar pattern was observed in the verification of sand-kaolin blends, with the slope of the SFCC graph rising as the amount of kaolin increased.

Keywords: construction material, frozen soil, geotechnical engineering, soil-freezing characteristic curve, unsaturated soil.

I. INTRODUCTION

Soil is a crucial element in construction, as its properties
significantly impact the stability and safety of any structure. Engineers often do not have the option to choose the type of soil at a construction site, as they must work with the subgrade conditions where the project is located. If the soil is not suitable, it can lead to a weak foundation, causing the building to settle or, in the worst case, collapse. Soil can be categorized based on its mechanical behavior, a field known as soil mechanics, which classifies soils into two states: saturated soils and unsaturated soils [17]. If soil is fully filled with water, it is categorized as saturated soil whereas soil that is not completely filled with water is classified as unsaturated soil [22], [36]. The difference between those two states of soil induced by different conditions is due to the contrast in the saturated and vadose zones [21], [35].

However, in long-frozen (permafrost) and seasonally frozen regions, the overall scenario is different as ice has an important role on the characteristics of soil. Therefore, it is significant to take into account frozen soil behavior when considering the unsaturated soils in cold areas such as Canada, Central Asia countries and Northern United States. For example, in Canada in the province of Quebec frost can penetrate the soil to depths of more than 1.5 m and its freezing indices range between 800 and 2000 degree-days [25]. Frost formation generally occurs in frost-susceptible soils that cause the generation of ice lenses and the heave surface that damages structures built on it. Changes in ice content and unfrozen water content substantially affect the mechanical and physical

properties of soil since unfrozen ice and water coexist in frozen soil areas [5].
Soil-freezing characteristics curve (SFCC) is the relationship between the unfrozen water content and temperature. To predict the frozen soil behavior, it is needed to estimate the SFCC. Previously, several empirical approaches were developed to model SFCC. At first, the SFCC was derived as a relationship between freezing temperature and pore radius, and the distribution function of SFCC was plotted by a perspective of exponential function [23], [24]. Another study estimated the SFCC by assuming that the shape of SFCC depends on pore size distribution of the soil. In addition, a mathematical expression for SFCC was proposed in the form of frequency distribution curve [32]. This research gives highly accurate and promising results as it has used unsaturated soils samples under extremely low temperature to determine the SFCC as those are crucial conditions for the intended research.
Soil-Water Characteristic Curves (SWCC) have been represented by various equations over the years, proposed by researchers like Brooks and Corey [7], van Genuchten [31], Fredlund and Xing [11], Zhai et al. [34] and Satyanaga et al. [27]. Among these, the eguation proposed in the study of Fredlund and Xing [11] is known for providing an excellent fit to experimental data [17]. However, the parameters in this equation do not correspond directly to the physical features of the shape of SWCC, making it challenging to determine unique values for these parameters.
Recognizing these limitations, Satyanaga et al. [26] developed a new SWCC equation based on the lognormal distribution. Their equation includes parameters that are directly related to the characteristics of the SWCC. Therefore, the lognormal distribution function was used in this study as the foundation for modeling the Soil-Water Characteristic Curve (SWCC).
The objective of this research is to develop a new mathematical equation for modeling SFCC in both coarse- and fine-grained soils. The scope of works involve the investigation on the relationship between the Soil Water Characteristic Curve (SWCC) and the Soil Freezing Characteristic Curve (SFCC) based on lognormal distribution of soil pores. Experimental tests were performed to validate the new model, which was also assessed using existing models and data from the literature. The proposed equation was then compared with previous SFCC models that are typically used for estimating SFCC.

II. LITERATURE REVIEW

Since the temperature is below 0°C in cold regions, the
freezing front penetrates downward into the soil. The freezing front evolves further due to an imbalance between heat supplied and removed when the pore water is frozen. As heat is transferred to the environment from the upper section, it undergoes freezing. In contrast, the lower section remains unfrozen; between them, there is a partially frozen layer of frozen fringe [28] (Fig.1). The thermodynamic mechanisms happening in the frozen fringe make contribution to the formation of negative pore water pressure [19]. The unfrozen water in small pores and the pore ice at subzero temperatures coexist in equilibrium within frozen soil [12].

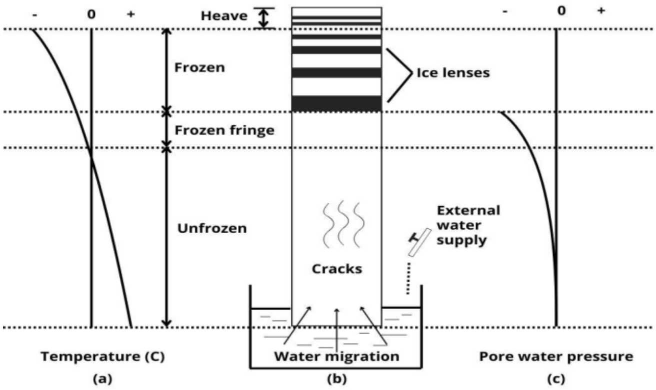

Fig. 1. Soil freezing a) temperature profile; b) soil profile; c) profile of pore water pressure.

The impact of frozen soil behavior on unsaturated soil properties is properly described by the correlation between SWCC and SFCC. At first it is required to understand that SWCC is related to drying or wetting processes as soil as it becomes unsaturated when subjected to dry conditions and consequently its voids are filled with water and air, while SFCC is closely related to freezing or thawing processes [14], [29]. Bi et all. [6] suggested that the water flow in the freezing front is similar to the water flow in the evaporative front. These studies are subjective to argument as they lack exact conclusions regarding the relationship between SWCC and SFCC. Along with this, the saturation degree of the soil is not specified. Koopmans and Miller [13] were the first researchers who estimated the SWCC and SFCC for sand, silt and coarse clay [15]. Consequently, Fig. 2 demonstrating the relationship between SWCC and SFCC could be plotted. The main similar trend between SWCC and SFCC was the increase in soil suction. Both graphs are divided into three zones. Air-entry value (AEV) that defines the transition between the boundary effect zone and transition zone in SWCC corresponds to ice-entry value (IEV) in SFCC. Here AEV refers to the suction value when air particles begin to enter big pores and similarly IEV refers to the subzero temperature when small ice crystals start to form in big pores, displacing some of the water [22]. Additionally, residual water content, θ_r, in SWCC parallels residual unfrozen water content, θ_r, which marks the shift from the transition zone to the residual zone.

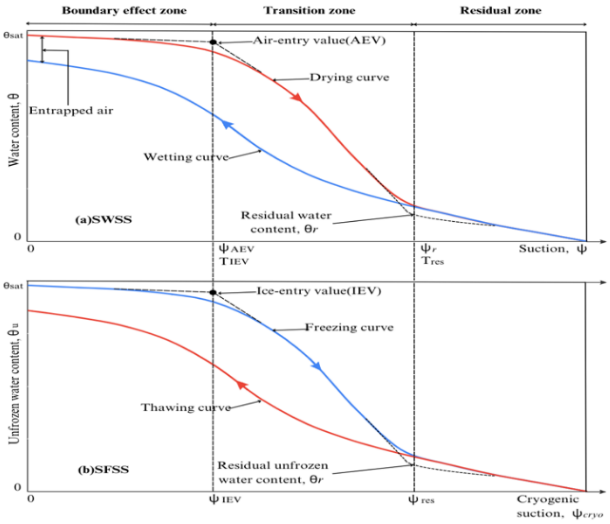

Fig. 2. The similarity between SFCC and SWCC.

SFCC is created mathematically from SWCC by linking the suction with temperature. Clapeyron equations are utilised at this stage for conversion between SWCC and SFCC. Brooks and Corey [7], van Genuchten [31], Fredlund and Xing [11], and Dall'Amico [9] provided some of these converted equations in which suction at the air-water layer in unsaturated unfrozen soil was substituted with suction at the ice-water interlayer in the saturated frozen soil [25]. To determine the saturation degree of ice and water during the freezing phase in unsaturated soils, Dall'Amico [9] integrated van Genuchten's SWCC model with the Clapeyron equation and derived an expression for total water content that is temperature independent:

$$\theta_l = \theta_{res} + (\varepsilon - \theta_{res})\left\{1 + \left[-aP_{w0} - a\frac{H_f\,\rho_w}{273.15}(T - \Delta T) \times H(T - \Delta T)\right]^n\right\}^{-m} \quad (1)$$

where a, n, and m are the van Genuchten [31] fitting parameters of SWCC model, H is the Heaviside function, ε is porosity and P_{w0} is pre-freezing pressure (M L^{-1} t^{-2}).
Sheshulov and Nieber [28] derived a formula for the relationship between unfrozen water content and sub-freezing temperature by incorporating the Clapeyron equation (Eq. 9) with Brooks and Corey SWCC equation and expressed it as the following equation:

$$\theta_u = \theta_{res} + (\theta_{sat} - \theta_{res})\left(\frac{-1L\rho_w T}{\psi_{IEV}\;273.15}\right)^{-1/b} \quad (2)$$

where ψ_{IEV} is ice-entry value (kPa), and b is the Brooks and Corey [7] model exponent of SWCC.

Azmatch et al. [4] approximated SFCC by using suction at the ice-water interface instead of the air-water contact used by Fredlund and Xing SWCC [11]. He has employed Clapeyron equation in Eq (9) and suggested the SFCC as [25]:

$$\theta_u = \frac{\theta_{sat}}{\left\{ln\left[2.718 + \left(\frac{L\rho_w}{a_f}ln\left(\frac{T+273.17}{T_0+273.15}\right)\right)^{n_f}\right]\right\}^{m_f}} \quad (3)$$

where a_f, m_f, and n_f, are the equation parameters of Fredlund and Xing [11] SWCC model.

Next equation for SFCC was suggested by Zhang et al. [37] in which Clapeyron equation (Eq.9) was integrated into the van Genuchten [31] SWCC model in the following manner:

$$\theta_u = \theta_{res} + (\theta_{sat} - \theta_{res})\left[1 + \left(a_{vg}L\rho_w ln\frac{T+273.17}{T_0+273.15}\right)^{n_{vg}}\right]^{-m_{vg}} \quad (4)$$

where a_{vg}, m_{vg}, and n_{vg} are the van Genuchten [31] fitting parameters of SWCC model.

The above listed equations by previous researchers did not properly account for freezing or thawing point. Bi et al., [6] developed the following equation (Eq.5) for SFCC that contains the freezing or thawing point in temperature.

$$\theta = \theta_r + (\theta_0 - \theta_r)\left[\frac{1}{1+\left(\frac{T}{T_{f/t}}-1\right)^n}\right]^m \quad for\ T < T_{f/t}$$

$$\theta = \theta_0 \quad for\ T > T_{f/t} \tag{5}$$

where $T_{f/t}$ is the freezing or thawing point and m=1-1/n

The motivation to develop the new equation stems from the need for a more physically relevant depiction of the Soil Freezing Characteristic Curve. Prior mathematical models, such as those by Dall'Amico [9], Sheshulov and Nieber [28], Azmatch et al. [4], and Zhang et al. [39], successfully integrated the Soil Water Characteristic Curve (SWCC) into the SFCC using the Clapeyron equation, however they didn't accurately consider the freezing or thawing points, nor did they fully capture the dynamic behavior of soil at different temperatures. These models depended mainly on fitting parameters rather than directly incorporating the physical phase transition between frozen and unfrozen states.

III. DEVELOPMENT OF NEW EQUATION FOR SFCC

The link between pressure and temperature in freezing soils is described by the Clapeyron equation, also known as the 'equilibrium equation' or the 'freezing temperature equation'. Numerous different distribution variants of this equation have been reported. Temperature is stated in degrees Celsius in this study, although it is represented in degrees Kelvin in many resources. Loch [18] developed a modified variant of the standard Clapeyron equation to apply on freezing soils using fundamental thermodynamic concepts by first incorporating the Gibbs-Duhem equations [15] to each phase before merging the terms:

$$\frac{P_{wf}}{\rho_w} - \frac{P_i}{\rho_i} = \frac{H_f}{273.15} \tag{6}$$

where T is the equilibrium freezing temperature (°C), H_f is the latent heat of fusion (L^2t^{-2}), P_{wf} and P_i are the equilibrium gauge pressures in partially frozen soil for the liquid water and ice phases ($M\ L^{-1}\ t^{-2}$), while ρ_i and ρ_w are the ice and water densities, respectively ($M\ L^{-3}$).

Loch [18] Eq. (7) was developed based on the theory from Schofield (1935) taking account a change in the equilibrium gauge pressure of water in frozen soil dP_{wf} because of the change in temperature dT. The equation is as follows:

$$\frac{dP_{wf}}{dT} = \frac{H_f\,\rho_w}{(T+273.15)} \tag{7}$$

Eq. (7) specifies that the equilibrium freezing temperature, and the pore water pressure are in balance.

In general, the Clapeyron equation is written without any differentials to integrate with respect to P_{wf} and T. By rearranging the Eq. (7) to eliminate pressure differential and using the 1st term of Taylor expansion into Eq. (7), Eq 8 is derived [15]:

$$P_{wf} \approx H_f \rho_w \frac{T}{273.15} \tag{8}$$

Clapeyron equation in Eq (8) can be simplified in terms of the suction and osmotic suction is assumed to be negligible according to Kurylyk and Watanabe [15], as illustrated in Equation (9) as follows：

$$\psi = -L\rho_w ln\frac{T+273.15}{T_0+273.15} \tag{9}$$

where ρ_w is density of water, ψ is suction (kPa), T is sub-freezing temperature (°C), L is the latent heat of fusion of water (L=334,000 J/kg), T_0 is normal freezing temperature (T_0 =0).

This new SFCC equation was developed from Satyanaga et al. [26] SWCC model that is given below in Eq (10):

$$\theta_w = C(\psi)\left(\theta_r + \left\{\theta_s - \theta_r\left(1 - (\beta)efrc\left(\frac{ln\left(\frac{\psi_a - \psi}{\psi_a - \psi_m}\right)}{S}\right)\right)\right\}\right) \tag{10}$$

where θ_w is volumetric water content, θ_r is residual volumetric content, θ_s is saturated volumetric content, β is 0 at the suction range at or before air-entry value, and 1 at any suction beyond air-entry value, ψ is matric suction in view (kPa), ψ_a is soil's air-entry value (kPa), ψ_m is matric suction at inflection point of the SWCC (kPa), S is standard deviation of the SWCC, ψ_r is matric suction corresponding to θ_r on the SWCC (kPa) and $C(\psi)$ is the correction factor

Satyanaga et al. [26] SWCC equation was developed into SFCC equation using the method of previous researchers as mentioned in Eq (3). The same to what Azmatch et al, [4] has done, the suction at air-water interface was changed by the suction at ice-water interface; therefore, the SFCC is represented in terms of unfrozen volumetric water content θ_u and the matric suction in Eq (10) is replaced by Clapeyron equation in Eq (9). Consequently, the final modified equation is expressed as follows:

$$\theta_u = C(\psi)\left(\theta_r + \left\{\theta_s - \theta_r\left(1-(\beta)efrc\left(\frac{ln\left(\frac{\psi_a - L\rho_w ln\frac{T+273.15}{T_0+273.15}}{\psi_a-\psi_m}\right)}{s}\right)\right)\right\}\right) \quad (11)$$

Eq (11) is the new proposed equation to estimate SFCC. As it was modified from Satyanaga [26] SWCC, SFCC of the sample soil using this equation can be determined if the SWCC of that sample soil is available.

The primary advantage of the proposed equation, compared to previous models, is its incorporation of freezing and thawing points, which enhances its accuracy in representing phase transitions in unsaturated frozen soils. Unlike other models that rely heavily on fitting parameters, this equation captures physical processes more effectively by directly linking temperature and phase changes. The newly proposed equation (Eq. 11) explicitly includes freezing/thawing points, leading to a more realistic depiction of soil water behavior during transitions. Additionally, the fitting parameters have clear physical significance, representing variables in the SWCC, which enhances the model's robustness and applicability across various soil conditions. This makes our equation both a better fit for experimental data and more predictive.

IV. RESULTS & DISCUSSIONS

To ensure the results were both reliable and meaningful, each soil sample—both natural and sand-kaolin mixtures—was tested three times under controlled freezing and thawing cycles. At each temperature step, measurements of unfrozen water content and suction were recorded.

The average values from these three trials were then used to construct the experimental SFCC curves. To assess how well the new SFCC model fit the experimental data, we used nonlinear regression analysis. The coefficient of determination (R^2) served as the main indicator of model accuracy, showing how closely the predicted curves matched the observed results. This comparison included several established models: Fredlund & Xing, van Genuchten, exponential, and power-law equations.

For the natural soil from Astana, the proposed SFCC model achieved an R^2 value of 0.999, demonstrating an excellent fit. A significance level of 0.05 ($p < 0.05$) was used for all curve-fitting analyses. The regression and statistical fitting were performed using Excel Solver and Python's SciPy library (curve_fit function)

A.Verification of the Proposed Equation with Astana Soil

Three trials of SFCC testing have been conducted in the laboratory experiments. Results for SFCC testing comprise experimental SFCC from the laboratory works and the theoretical results from mathematical equations for SFCC. The Experimental SFCC was generated by employing the recordings of temperature and suction taken from the laboratory. For the theoretical SFCC, mainly Fredlund and Xing [11] and Satyanaga et al. [26] equations were utilized. For some theoretical SFCCs derived using power function, exponential equations and van Genuchten [31] equations for the measurement of unfrozen water content were used. After compiling all the results from the experimental SFCC, they were compared with the theoretical SFCC to verify if they correspond to each other.

The new SFCC equation as modified from Satyanaga et al [26] offers a novel dimension to the research. The applicability and robustness of the proposed equation are demonstrated by the verification stages, which include testing on Astana soil, sand-kaolin combinations, and a wider SFCC database by Devoie et al. [10] which has a Repository of 100+ Years of Measured SFCCs. This new SFCC equation provides a potential option for forecasting SFCCs and an additional tool set available for soil freezing characterisation.

As laboratory data for SFCC was available from the SFCC experiment and the data were best fitted using the newly proposed equation to check the agreement between the prediction using the proposed equation and the experimental data. As new SFCC equation was modified from Satyanaga et al [26] for modelling SWCC, its application requires SWCC variables such as θ_r residual volumetric content, ψ_{AEV} soil's air-entry value, ψ_m matric suction at inflection point, and ψ_r matric suction corresponding to θ_r on the SWCC. For the Astana soil sample, the following parameters were used: θ_s=0.251, θ_r=0.035, ψ_{AEV}=1000*10^3 Pa, ψ_m=1243.698 *10^3, ψ_r=10000*10^3, and σ =0.958. The resulting SFCC is shown in Fig. 3, demonstrating a strong agreement between the model and experimental data, validating the predictive capability of the proposed equation for fine-grained soils like Astana clay.

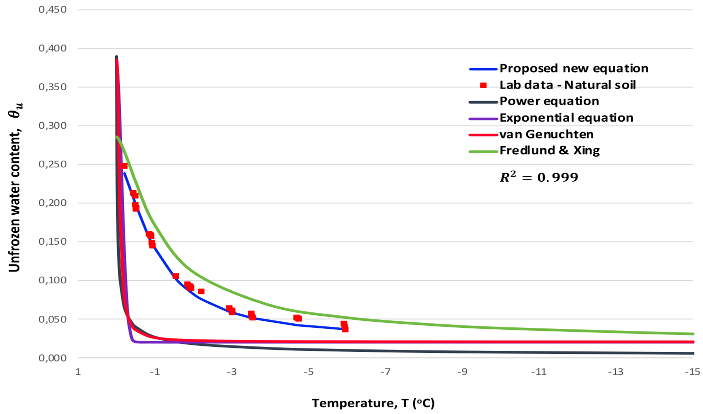

Fig. 3. Comparison of predicted SFCC from different equations and the experimental data for the Astana soil

B. Verification of the new proposed SFCC equation with database of various SFCCs

Finally, the developed SFCC equation was verified using the SFCCs database containing measured SFCCs for various types of soils over 100 years of measurements [10]. This SFCC metadata contains information on which kind of source SFCC is, method of SFCC testing and types of soils used in experiment. As the metadata has lots of SFCCs, the new equation was tested by using a selection of six SFCC data. However, this SFCC database had one drawback that are some of the soils' composition are unknown. Therefore, type of soils for all verified SFCCs from this database have title "N/A", meaning that name of the soil is Not Available. The final best fitted metadata SFCCs with the new proposed equation are shown in Fig. 4. It can be observed from these SFCC graphs that the new mathematical equation could model the SFCC sufficiently well.

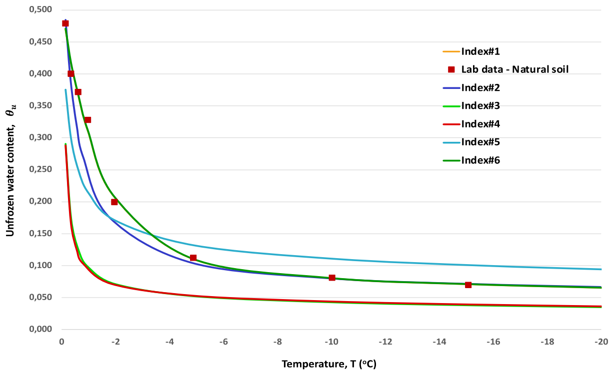

Fig. 4. SFCCs with proposed equation for soils from database

C. Verification of the new proposed SFCC equation by sand-kaolin mixture

The next verification was performed using the available SWCC data of sand-kaolin mixtures from the previous theoretical SFCCs part in section 5.2. In this case, SWCC parameters from Satyanaga et al [26] equation was again applied to each sand-kaolin mixture compositions. The variables are given in Table I.

Table I. Parameters from Satyanaga et al. [26] SWCC equation for sand-kaolin mixtures

Sand-Kaolin Mix	θ_s	θ_r	ψ_{AEV} ($\times 10^3$ Pa)	Ψ_m ($\times 10^3$ Pa)	σ	Ψ_m ($\times 10^3$ Pa)
90%S–10%K	0.25	0.02	1	60	5.0	6000
80%S–20%K	0.652	0.02	20	300	2.0	1000
70%S–30%K	0.296	0.02	7	150	4.0	4000
60%S–40%K	0.386	0.05	10	100	2.25	1000
50%S–50%K	0.455	0.02	1	21	2.2	300
40%S–60%K	0.64	0.0	4	40	1.8	400

30%S–70%K	0.635	0.04	8	130	2.0	1000
20%S–80%K	0.652	0.05	20	300	2.0	3000
10%S–90%K	0.71	0.05	100	2000	3.5	10000

After applying those parameters in Table I, nine SFCCs with new equation were generated as illustrated in Fig. 5. From these figures, it can be concluded that as the percentage of kaolin in the mixture increases the SFCC line becomes steeper for the temperature below -1°C. To sum up, as mixture becomes more fine-grained, the slope in SFCC graphs becomes steeper for the sand- kaolin mixtures as illustrated by the newly proposed SFCC equation.

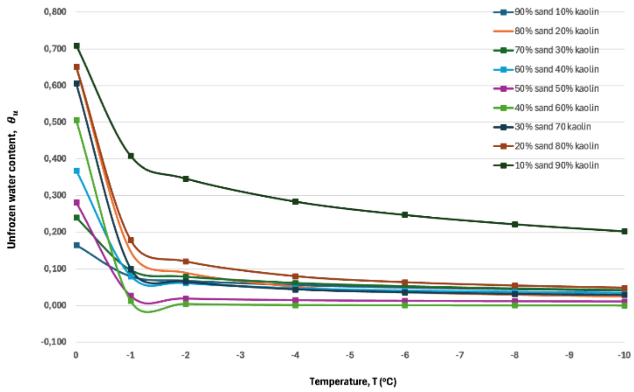

Fig. 5. SFCCs with new proposed equation for sand-kaolin mixtures of various composition

D. DISCUSSION

The behavior of the Soil Freezing Characteristic Curve (SFCC) during the freezing-thawing cycle can be explained by three phases [38]. The first phase occurs during the freezing process, where the SFCC typically remains unchanged until the temperature reaches 0 °C. When the temperature drops below 0 °C, the SFCC decreases significantly due to the formation of a substantial amount of ice in the soil sample, marking the second phase. In the third phase, the SFCC stabilizes because further temperature declines at a specific subzero point does not cause a significant decrease in the unfrozen water content. A similar trend can be observed during the thawing process. Due to the freezing-thawing hysteresis effect, the unfrozen water content in the freezing branch of the SFCC is higher than in the thawing branch at subzero temperatures [30]. This hysteresis effect is more noticeable at higher temperatures (-5°C to 0°C). Study of Li et al. [16] illustrated similar results.

The actual experimental data were used to evaluate the new SFCC model. As shown in Figure 5, the experimental data of SFCC of Astana soil match the best-fitting line of the new equation. The newly proposed model has been shown to accurately represent the SFCC variation during the freezing process. When comparing the best fit of the experimental SFCC using the Fredlund & Xing [11] equation with the new model as shown in Fig. 3, the new SFCC equation fits the Astana soil better than the Fredlund & Xing [11] SFCC. Nevertheless, both SFCCs exhibit similar patterns. Their boundary effect zone is the same, meaning that from 0 °C to -0.5 °C, they gradually decline where there is no significant ice. After -0.5 °C, the transition zone begins, and both SFCCs experience a sharp drop in unfrozen water content as suction increases rapidly. Furthermore, at - 4 °C, there is a residual zone where fluctuations in the unfrozen water content are minimal despite substantial variations in temperature or suction.

The new model was also successfully verified using sand-kaolin mixtures. The graphical analysis indicates a clear pattern where, as the amount of kaolin in the mixture increases, the SFCC graph becomes steeper. This feature remains unchanged when applying both the Fredlund & Xing and Satyanaga SFCC models. Slope angles of both models were calculated in relation to their SWCC air-entry values as shown in Table II. As evident from this table, in general, as the sand content in the mixture decreases, the slopes also decrease. This can be attributed to the fact that as the soil sample becomes finer, the slope in the SFCC graphs becomes steeper because of a larger specific surface area of soil particle and the higher fine content allows the soil particle to retain more unfrozen water through larger adsorptive pressures [23]. Based on these findings, it can be concluded that the new SFCC model accurately simulated SFCC fluctuations during the freezing process.

Table II. Comparison of slope angles of SFCCs and air-entry values of SWCC

Soil	SWCC Air-Entry Value ψ_{AEV} (kPa)	SFCC Slope (F&X Model)	SFCC Slope (Proposed Model)
90%S–10%K	1000	0.0107	0.0087

80%S–20%K	20000	0.0218	0.0398
70%S–30%K	7000	0.0049	0.0135
60%S–40%K	10000	0.0091	0.0202
50%S–50%K	1000	0.0063	0.0151
40%S–60%K	4000	0.0108	0.0274
30%S–70%K	8000	0.0175	0.0344
20%S–80%K	20000	0.0209	0.039
10%S–90%K	100000	0.0149	0.0382

CONCLUSION

This research investigated the essential characteristics of unsaturated and frozen soils through the analysis of the Soil Water Characteristic Curve (SWCC) and Soil Freezing Characteristic Curve (SFCC). We examined how different soil types influence these characteristics by applying mathematical SFCC models to various soils, including Astana soil and sand-kaolin mixtures, as well as SFCC envelopes derived from the Unified Soil Classification System (USCS). Notably, fine-grained soils demonstrated a higher unfrozen water content at temperatures below -10°C, attributed to their enhanced capacity for water retention.
The development of a new SFCC equation proved to be a significant advancement, effectively capturing the variations in SFCC during the freezing process. When compared to experimental data from Astana soil, this new model exhibited a better good performance as compared to the Fredlund & Xing [11] in modelling SFCC. A similar pattern was observed in sand-kaolin mixtures, where increased kaolin content led to a steeper slope in the SFCC graph, consistent with both the Fredlund & Xing and Satyanaga SFCC models.
In conclusion, this study provides valuable insights into the intricate relationships between soil type, freezing-thawing behavior, and the accuracy of SFCC models. These findings hold potential for advancing soil engineering practices, particularly in cold climates, by offering a detailed understanding of the interactions between soil water dynamics and freezing characteristics.

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