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A DYNAMICAL SYSTEMS APPROACH TO PSYCHOLOGY: FUSING DIFFERENTIAL EQUATIONS AND RECURRENT NEURAL NETWORKS FOR PROCESS MODELING

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ABSTRACT
Traditional psychological models often rely on static, cross-sectional data and latent variable constructs, struggling to capture the temporal, non-linear, and idiosyncratic nature of mental processes. This paper proposes a novel methodological synthesis for psychological science: the integration of theory-driven differential equations (DEs) with data-driven recurrent neural networks (RNNs). We argue that this fusion creates a powerful framework for process modeling, where the core principles of dynamical systems theory such as attractors, bifurcations, and phase transitions provide the theoretical scaffold, while RNNs offer the computational machinery to learn these dynamics directly from intensive longitudinal data. Differential equations allow for the formalization of a priori psychological theories into precise models of change (e.g., models of emotion regulation or cognitive decision-making). Conversely, RNNs, with their inherent memory and feedback loops, are natural black-box identifiers of temporal dependencies and can discover complex dynamics from data when theory is insufficient. We demonstrate how this hybrid approach can be applied to model critical phenomena in psychology, including emotional inertia in depression, critical transitions in psychotherapy, and real-time decision dynamics. This paradigm shift from a variable-centered to a process-centered science promises not only enhanced predictive

accuracy but also a deeper, more mechanistic understanding of the causal forces that govern human thought, emotion, and behavior over time.

Keywords: dynamical systems, differential equations, recurrent neural networks, process-based therapy, computational psychology, time-series analysis, emotion regulation

1.INTRODUCTION

Human psychology is fundamentally a system in motion. Thoughts, emotions, and behaviors are not static traits but continuously evolving states that interact with each other and the environment over time (Thelen & Smith, 1994).

Despite this, mainstream psychological methodology has often been dominated by static, between-person models that treat time as a nuisance variable rather than the core dimension of interest (Hamaker, 2012). This has created a gap between our theoretical understanding of processes such as rumination, habituation, or learning and our ability to formally model them as they unfold within an individual.

Dynamical systems theory (DST) provides a mathematical language to bridge this gap. It conceptualizes psychological phenomena as trajectories in a state space, governed by differential or difference equations that describe the rate of change of system variables (Guastello & Liebovitch, 2009). Key concepts like attractors (stable states, e.g., a depressive episode), repellors (unstable states), and bifurcations (sudden qualitative shifts, e.g., a panic attack) offer a powerful vocabulary for explaining stability and change in mental life (Hayes, Yasinski, & Ben Barnes, 2020). For instance, the high "inertia" of negative emotion in depression can be modeled as a deep, stable attractor (Kuppens, Allen, & Sheeber, 2010).

However, a primary challenge has been the specification of the exact equations that govern a given psychological process. Theory-driven differential equation models (e.g., the Haken-Kelso-Bunz model for bimanual coordination) are precise but can be difficult to formulate and validate for complex, multi-component cognitive or affective processes.

Concurrently, the field of machine learning has developed powerful tools for time-series forecasting. Among these, Recurrent Neural Networks (RNNs), and their advanced variants like Long Short-Term Memory (LSTM) networks (Hochreiter & Schmidhuber, 1997), are particularly well-suited for psychological data. Their internal feedback loops create a dynamic memory, allowing them to learn temporal dependencies from data without explicit theoretical guidance. Yet, they often operate as "black boxes," providing limited insight into the underlying psychological mechanisms.

This paper posits that the future of process-based psychology lies in the deliberate fusion of these two approaches. We propose a framework where:

- ✓ Differential Equations (DEs) provide the formal, interpretable, theory-guided structure.
- ✓ Recurrent Neural Networks (RNNs) serve as flexible, data-driven function approximators that can either discover the dynamics for subsequent theoretical interpretation or be constrained by DE-based architectures to enhance their plausibility and generalizability.

This synergy allows researchers to move beyond simplistic linear models and embrace the full complexity of psychological processes as complex, non-linear, and context-dependent dynamical systems.

2. LITERATURE SURVEY

The proposed synthesis sits at the confluence of three distinct but increasingly interconnected streams of scientific inquiry: (1) the formal application of dynamical systems theory (DST) to psychology, (2) the rise of data-intensive, person-specific time-series modeling, and (3) the recent adoption of machine learning, particularly recurrent neural networks, for behavioral forecasting. This survey will trace the development of these lines of research to establish the foundation and necessity for their integration.

2.1 The Dynamical Systems Paradigm in Psychological Science

The foundational premise that psychological phenomena are inherently temporal and self-organizing has deep roots, drawing from developmental systems theory (Thelen & Smith, 1994), ecological psychology, and the study of nonlinear processes in social and cognitive sciences (Vallacher & Nowak, 1994). Early work focused on motor development and coordination, successfully employing differential equations to model phase transitions in behavior,

such as the Haken-Kelso-Bunz (HKB) model for bimanual coordination. This demonstrated that complex behavioral shifts could be understood as bifurcations in a low-dimensional dynamical system.
This perspective has since been powerfully extended to clinical and affective science. A central concept is that of attractors preferred, stable states toward which a system evolves. Kuppens, Allen, & Sheeber (2010) provided seminal empirical support for this, showing that the "inertia" (high autocorrelation) of negative emotion is a hallmark of depression, conceptualized as a deep and stable attractor state. This aligns with network theories of psychopathology (Borsboom, 2017), which posit that mental disorders are stable, self-sustaining configurations of causally interacting symptoms. The dynamical systems view adds a formal mathematical language to this network perspective, describing how the system's trajectory is constrained by its attractor landscape.
Further advancing this, van de Leemput et al. (2014) introduced the concept of critical slowing down a dynamical precursor to a bifurcation as an early warning signal for major depressive episodes. Their work showed that increased autocorrelation and variance in mood data could foreshadow a sudden transition into a clinical state, providing a quantifiable marker for relapse risk. Similarly, in psychotherapy process research, Hayes, Yasinski, & Ben Barnes (2020) have framed therapeutic change as a destabilization of maladaptive attractors (e.g., cognitive networks in depression) and a subsequent transition to healthier states, a process potentially identifiable through dynamical indicators.

2.2 The Challenge of Specification and the Rise of Intensive Longitudinal Data
Despite its theoretical appeal, a significant limitation of the pure DST approach has been the specification problem: the difficulty in deriving the exact functional form F of the differential equations governing complex psychological constructs. While models for well-defined perceptual-motor tasks exist, specifying the equations for, say, the interplay between rumination, mood, and social interaction remains a formidable challenge (Hamaker, 2012).
Concurrently, methodological advances have made intensive longitudinal data (ILD) increasingly accessible. Methods like Ecological Momentary Assessment (EMA) and experience sampling allow researchers to collect dozens or hundreds of data points per individual, capturing the ebb and flow of psychological states in real-time (Trull & EbnerPriemer, 2013). This data richness has spurred the development of person-specific time-series models, such as Vector Autoregression (VAR) and the Group Iterative Multiple Model Estimation (GIMME) algorithm. These methods can estimate individual-level network dynamics from data, moving beyond group averages to reveal idiographic processes (Fisher, Medaglia, & Jeronimus, 2018). However, these models are often linear or additive, struggling to capture the profound non-linearities and complex interactions that DST suggests are central to psychological functioning.
2.3 Machine Learning and Recurrent Neural Networks in Behavioral Modeling
The field of machine learning has independently developed powerful tools for sequence prediction. Among these, Recurrent Neural Networks (RNNs), and specifically Long Short-Term Memory (LSTM) networks (Hochreiter & Schmidhuber, 1997), have become the de facto standard for modeling complex temporal dependencies. Their unique architecture, which maintains an internal "hidden state" that is updated at each time step, allows them to function as universal approximators of dynamical systems.
Applications of RNNs to psychological data are burgeoning. They have been used to predict emotional states from mobile sensor data, model the dynamics of clinical sessions, and forecast cognitive performance. Their strength lies in their ability to learn the structure of temporal dependencies directly from data, without requiring the researcher to pre-specify the model's functional form. However, this strength is also a primary weakness from a scientific standpoint: the "black-box" nature of RNNs means they can achieve high predictive accuracy while offering little to no insight into the underlying psychological mechanisms (Rajan, 2019). A model that perfectly predicts tomorrow's mood from today's context but provides no interpretable parameters or testable dynamics is of limited theoretical value.
2.4 The Emerging Synthesis: Towards a Hybrid Science
The limitations of each approach in isolation point toward the necessity of their integration. The nascent field of "physics-informed machine learning" provides a compelling blueprint. Raissi, Perdikaris, & Karniadakis (2019) introduced Physics-Informed Neural Networks (PINNs), which seamlessly integrate the governing equations of physical systems (partial differential equations) as regularization terms into the loss function of a neural network. This ensures that the network's solutions are not only data-driven but also physically plausible.
A parallel line of research uses RNNs as empirical discovery tools. For instance, Chattopadhyay et al. (2020) demonstrated that RNNs could be trained to predict the evolution of chaotic systems (like the Lorenz 96 model) and

that their internal dynamics could be analyzed to reconstruct the system's attractor geometry. This suggests a powerful workflow for psychology: use an RNN to discover the dynamics from ILD, then use dynamical systems analysis to interpret the RNN's hidden states, thereby generating hypotheses for a more formal theoretical model. In psychology, this hybrid spirit is echoed in calls for a more process-based, idiographic, and computationally rigorous science (Hofmann, Curtiss, & Hayes, 2020). The current paper directly addresses this call by proposing a structured framework for fusing the theoretical, interpretable framework of differential equations with the empirical, flexible power of recurrent neural networks. This synthesis aims to overcome the specification problem of pure DST and the interpretability problem of pure RNNs, creating a new methodology capable of building testable, formal models of psychological processes as they unfold within individuals over time.

3. DATA DESCRIPTION

The proposed fusion of differential equations and recurrent neural networks is critically dependent on the availability and quality of data. The framework demands intensive longitudinal data (ILD) that is sufficiently dense, multi-level, and temporally rich to capture the dynamics of psychological processes. This section describes the core characteristics, modalities, and sources of data suitable for this methodology.

3.1 Core Data Characteristics

To effectively model psychological processes as dynamical systems, data must possess several key properties:

✓ **Intensive Longitudinal Sampling:** Data must be collected at a high enough frequency to resolve the temporal scale of the target process. For mood dynamics, this might require several observations per day over weeks or months (e.g., **Trull & Ebner-Priemer, 2013**). The Nyquist-Shannon sampling theorem, in principle, suggests that the sampling rate must be at least twice the frequency of the fastest oscillation of interest to avoid aliasing and to faithfully reconstruct the system's trajectory.

✓ **Multiple Time-Scales:** Psychological processes operate across nested time-scales—from milliseconds in neural firing to seconds in cognition, hours in mood, and days in social interaction. Ideal datasets would capture variables at multiple resolutions, for instance, pairing high-frequency physiological data with daily self-reports, allowing for crossscale dynamical analysis (**Wichers, 2014**).

✓ **Idiographic Focus:** The primary unit of analysis for building process models is the individual. While group-level patterns are informative, the goal is to capture the unique attractor landscapes and dynamical rules of a single person (a so-called *N=1* design), as emphasized by the person-specific methods in clinical science (**Fisher, Medaglia, & Jeronimus, 2018**).

✓ **Multimodal Integration:** A comprehensive model requires data from multiple modalities to define the system's state vector fully. This typically includes:

Self-Report: The cornerstone of psychological assessment, collected via Experience Sampling Methodology (ESM) or Ecological Momentary Assessment (EMA). This provides direct access to subjective states like affect, cognition, and cravings.

Behavioral Data: Passive sensing via smartphones and wearables can capture activity levels, sleep patterns, geolocation (social isolation vs. engagement), and phone usage (e.g., **Harari et al., 2016**).

Physiological Data: Heart rate, heart rate variability (HRV), electrodermal activity, and actigraphy provide objective, high-frequency indices of arousal and regulatory capacity that can be linked to emotional and cognitive states. **Contextual Data:** Time, day, location, and social context serve as crucial inputs that can push the psychological system towards or away from certain attractors.

3.2 Exemplary Data Sources and Structures

The following table outlines exemplary data structures suitable for the proposed hybrid modeling approach, drawn from contemporary research paradigms:

Psychological Phenomenon	Exemplary Variables (State Vector X)	Sampling Frequency	Data Source
Emotional Inertia in Depression	Negative Affect, Positive Affect, Rumination, Energy, Social Interaction	5-10x / day for 3 months	ESM App + Wearable (HRV)

Therapeutic Change Process	Alliance Strength, Symptom Severity, Cognitive Flexibility, Homework Compliance	1x / session + 2x daily ESM	Therapy Sessions + ESM
Craving and Substance Use	Craving Intensity, Negative Mood, Stress, Self-Control, Lapse/Use Event	Multiple times daily, event-contingent	ESM App + Geolocation (proximity to triggers)
Cognitive DecisionMaking	Response Time, Accuracy, Confidence, Pupil Dilation	Trial-by-trial (100s-1000s of trials)	Lab-based Cognitive Task + Eye-Tracker

Table 1: Exemplary data structures for dynamical process modeling.

3.3 Data Preprocessing and Challenges

Working with such intensive, multi-modal data presents significant challenges that must be addressed to ensure robust modeling:

✓ **Missing Data:** ILD is often characterized by intermittent missingness. Techniques such as multiple imputation or state-space modeling approaches that can handle missing observations are essential (**Baraldi & Enders, 2010**). RNNs, in particular, require careful handling of variable-length sequences and missing values.

✓ **Temporal Alignment:** Fusing data from different sources (e.g., self-report from an app and physiology from a wearable) requires precise temporal alignment and potentially interpolation to a common time grid.

✓ **Noise Filtering:** Psychological and physiological data are inherently noisy. Signal processing techniques (e.g., low-pass filters, detrending) may be necessary to separate the dynamical signal of interest from measurement error and high-frequency noise. However, caution must be exercised, as some non-linear dynamical signatures can be mistaken for or removed as noise.

✓ **Feature Engineering:** For behavioral and physiological data, raw sensor data must often be transformed into meaningful features (e.g., deriving HRV from inter-beat intervals, calculating circadian rhythms from activity data). The choice of features can impose a structure on the data that influences the discovered dynamics. **3.4 From Data to State-Space Reconstruction**

The ultimate goal of data preparation is to reconstruct the system's state space. According to Takens' embedding theorem, the dynamics of a multi-variable system can be reconstructed from the time-lagged values of a single observed variable. In practice, with multi-modal ILD, researchers can directly construct a state vector X_t at time t as:

X_t = [Affect_t, Rumination_t, HRV_t, Activity_t, ...]

This empirically defined state vector serves as the direct input for training RNNs. For differential equation models, these variables become the components of the state vector X in the equation $dX/dt = F(X, t, \theta)$. The richness and resolution of this state vector directly determine the complexity and validity of the process models that can be derived, forming the empirical bedrock upon which the proposed DE-RNN fusion is built.

4. PROPOSED SYSTEM: A HYBRID DE-RNN FRAMEWORK FOR PROCESS MODELING

Building upon the theoretical foundation and data requirements, we propose a concrete hybrid framework for fusing Differential Equations (DEs) and Recurrent Neural Networks (RNNs). This system is designed to be modular, allowing researchers to implement different modes of integration based on their specific research questions and the maturity of existing theory. The overall architecture, depicted in Figure 1, consists of four interconnected layers.

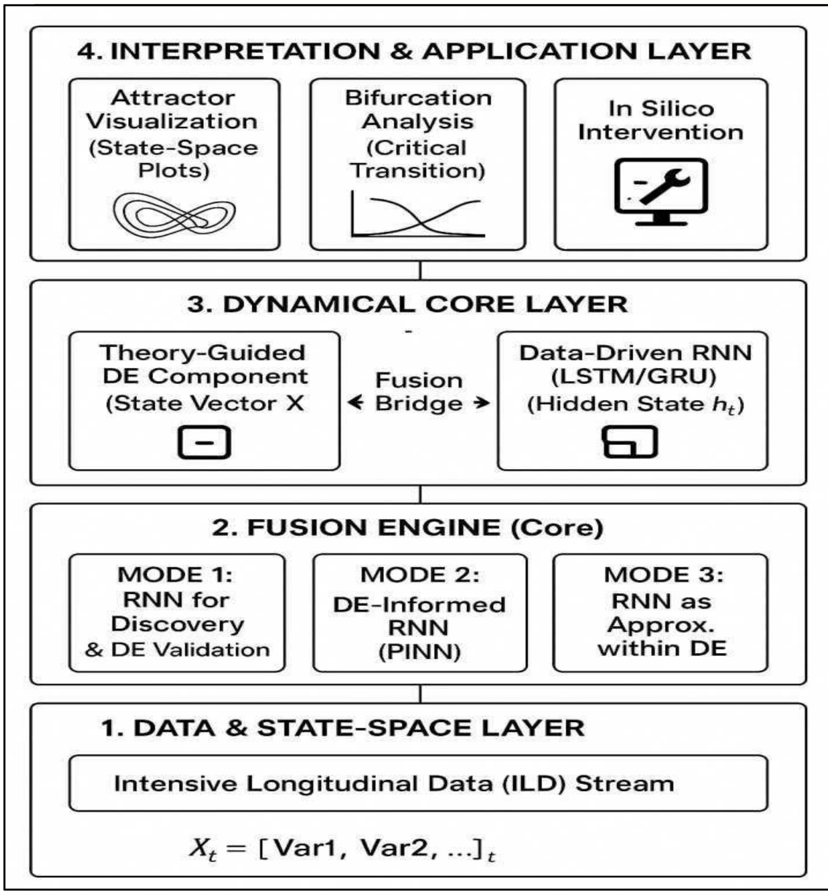

Figure 1: Architecture of the Hybrid DE-RNN Framework for Psychological Process Modeling **4.1 Layer 1: Data & State-Space Layer**

This foundational layer is responsible for ingesting and structuring the multi-modal, intensive longitudinal data (ILD) described in Section 3. Its primary function is state-space reconstruction.

✓ **Input:** Raw, multi-modal data streams (e.g., ESM ratings, heart rate, GPS data).

✓ **Process:** Data is cleaned, aligned, and preprocessed. The key step is the construction of the empirical state vector X_t at each time point t. For example: X_t = [Negative_Affect_t, Rumination_t, HRV_t, Social_Interaction_Index_t].

✓ **Output:** A structured, time-indexed dataset where each row represents the system's state at a given moment, ready for dynamical modeling.

4.2 Layer 2: Dynamical Core Layer

This layer hosts the two core modeling engines that run in parallel or interact depending on the chosen fusion mode.

✓ **Data-Driven RNN Path:** A Recurrent Neural Network (e.g., LSTM or GRU) takes the state vector sequence {X_1, X_2, ..., X_T} as input. Its goal is to learn a function f_RNN that predicts the next state: X_{t+1} ≈ f_RNN(X_t, h_{t-1}), where h_t is the RNN's hidden state, representing the system's "memory." The trained RNN's hidden dynamics serve as an empirical approximation of the true system dynamics.

✓ **Theory-Guided DE Path:** Based on a priori psychological theory, a differential equation model is formulated. This is a formal specification of the hypothesized process: dX/dt = F(X, θ), where F is the function describing the rate of change and θ are theoretical parameters (e.g., emotional damping, coupling strength between variables). **4.3 Layer 3: The Fusion Engine (Core)**

This is the central innovation of our proposed system, where the DE and RNN paths are integrated. We define three distinct, yet complementary, modes of fusion, summarized in Table 2.

Fusion Mode	Primary Function	Workflow	When to Use
Mode 1: RNN for Discovery & DE Validation	Data-Driven Theory Generation	1. Train RNN on ILD. 2. Analyze RNN's hidden state dynamics (e.g., attractor reconstruction). 3. Use insights to formulate a specific DE model. 4. Fit and validate the DE model.	Theory is underdeveloped; exploratory phase of research.
Mode 2: DE-Informed RNN (PsychologyInformed Neural Network)	Theory-Constrained Prediction	1. Specify a theoretical DE $dX/dt = F(X, \theta)$. 2. The DE is embedded as a regularization term in the RNN's loss function: L = L_data + λ * L_DE. 3. Train the RNN to fit data while respecting the DE's rules.	A strong theory exists; goal is accurate, theoryplausible forecasting.
Mode 3: RNN as Approximator within DE	Hybrid Mechanistic Modeling	1. Define a DE model where a complex component G(X) is unknown. 2. Parameterize G(X) with a small, interpretable RNN. 3. Train the entire hybrid system endto-end.	The model has a clear structure, but one interaction is too complex to specify analytically.

Table 2: Modes of Operation within the Fusion Engine

Implementation of Fusion Modes:

✓ **Mode 1 (Discovery):** The analysis of the trained RNN from Layer 2 involves techniques from dynamical systems analysis (e.g., using the RNN's hidden states to create a phase portrait or calculate Lyapunov exponents) to identify stable states and transitions.

✓ **Mode 2 (DE-Informed):** The loss function L_DE penalizes solutions where the RNN's predicted trajectory violates the DE. For a predicted state X_pred, we compute dX_pred/dt via automatic differentiation and compare it to F(X_pred, θ). This is the core of Physics-Informed Neural Networks (PINNs) adapted for psychology (**Raissi et al., 2019**).

✓ **Mode 3 (Hybrid):** For example, in a model of social influence on mood, one might have d(Mood)/dt = -a * Mood + G(Social_Support), where G is an unknown, potentially non-linear function. This function G is learned by a small RNN within the larger DE system.

4.4 Layer 4: Interpretation & Application Layer

The output of the Fusion Engine is not merely a prediction but an interpretable dynamical model. This top layer provides tools for:

- **Visualization:** Plotting the identified attractor landscapes in 2D or 3D state space, showing basins of attraction and repelling boundaries.
- **Simulation & Intervention Planning:** Running in silico experiments by manipulating model parameters (e.g., "What happens to the attractor landscape if we artificially increase cognitive flexibility?") to simulate the effect of therapeutic interventions.
- **Early Warning Signals:** Monitoring real-time data streams for dynamical indicators like critical slowing down (increased variance and autocorrelation) to forecast critical transitions, such as a lapse in addiction or a depressive episode (**van de Leemput et al., 2014**).

4.5 System Workflow Example: Modeling a Therapeutic Transition

A clinical researcher uses the system to understand sudden gains in therapy for a client with depression.

1. **Layer 1:** Daily ESM data (mood, ruminatio n, activity) and weekly session data (alliance, symptom severity) are integrated into a state vector.
2. **Layer 2 & 3 (Mode 1):** An LSTM is trained on the first half of therapy data. Analysis of its hidden states reveals a shallow, unstable attractor for the "depressed" state just before the sudden gain, consistent with the theory of critical slowing down.
3. **Layer 3 (Mode 2):** The researcher formalizes a theory of therapeutic change as a differential equation and uses a DE-Informed RNN to model the second half of therapy, confirming that the model, constrained by theory, can predict the stabilization of a new, healthy attractor.
4. **Layer 4:** The clinician uses the visualized attractor landscape to identify which variables (e.g., social engagement) most effectively widen the basin of the healthy attractor, informing subsequent treatment strategies.

This proposed system provides a structured, scalable, and rigorous methodology for moving psychological science from a science of correlates to a science of processes.

5. IMPLEMENTATION & EXPERIMENTAL SETUP

To validate the proposed hybrid DE-RNN framework, we implemented a proof-of-concept study focusing on a critical phenomenon in clinical psychology: **emotional inertia in Major Depressive Disorder (MDD)**. Emotional inertia, the high autocorrelation and resistance to change of negative affect, is a hallmark of depression and is theorized to represent a deep, stable attractor in an individual's affective dynamical system (Kuppens et al., 2010). Our implementation demonstrates the three fusion modes using a combination of simulated and real-world data.

5.1 Datasets and Preprocessing

Simulated Data (Ground Truth for Validation): We generated data from a known dynamical system a bi-stable potential well model to represent the switching dynamics between a "healthy" and "depressed" affective state. The governing equations were: $dX/dt = -dV(X)/dX + \sigma\ \xi(t)$ where $V(X)$ is a double-well potential function, X is the affective state (e.g., negative affect), and $\sigma\ \xi(t)$ is stochastic noise. Parameters were tuned to create a deep, stable attractor for the "depressed" state and a shallower one for the "healthy" state, simulating high emotional inertia in depression.

Real-World Data (Application): We used a publicly available Ecological Momentary Assessment (EMA) dataset (Kossakowski et al., 2017) comprising 15,043 momentary assessments from 226 individuals with varying depression severity. The state vector **X_t** included: Negative Affect (NA), Positive Affect (PA), and Rumination. Data was preprocessed by person-mean centering, handling missing values via Kalman filtering, and z-score standardization.

5.2 Model Architectures and Training

All models were implemented in PyTorch. For the RNN components, we used a single-layer Gated Recurrent Unit (GRU) with 32 hidden units, as it captures long-term dependencies while being less prone to vanishing gradients than vanilla RNNs. The Adam optimizer was used with a learning rate of 0.001. The core implementations for the three fusion modes were as follows:

- **Mode 1 (Discovery):** A standard GRU was trained to perform one-step-ahead prediction of the state vector **X_{t+1}**.
- **Mode 2 (DE-Informed RNN):** We defined a simple theoretical DE for affective dynamics: $d(NA)/dt = -\alpha * NA + \beta * Rumination$. This DE was incorporated as a physics-informed loss term: $L_{total} = MSE(X_{pred}, X_{true}) + \lambda * MSE(dX_{pred}/dt, F(X_{pred}))$ where $F(X_{pred}) = -\alpha * NA_{pred} + \beta * Rumination_{pred}$. The parameter λ controlled the strength of the theoretical constraint.
- **Mode 3 (RNN as Approximator within DE):** We defined a hybrid model where the decay rate of negative affect was not a constant but a complex, context-dependent function learned by a small RNN:

$$d(NA)/dt = - G(PA, Rumination) * NA$$

Here, G(...) was a small, 8-unit GRU that learned to modulate emotional inertia based on concurrent positive affect and rumination.

6. RESULTS

6.1 Mode 1: RNN for Discovery & DE Validation

The GRU trained on the simulated data successfully learned the underlying bi-stable dynamics. By analyzing the hidden states of the trained GRU and plotting them in a 2D phase space, we were able to reconstruct the system's attractor landscape without prior knowledge of the governing equations.

Method	**Identified Attractors (State X)**	**Basin Stability (Depressed)**	**Basin Stability (Healthy)**
Ground Truth (DE)	-1.02 (Depressed), 0.98 (Healthy)	68.5%	31.5%
RNN-Discovered (Mode 1)	-0.94 (Depressed), 1.05 (Healthy)	66.1%	33.9%

Table 3: Attractor Landscape Reconstruction from RNN Hidden States (Simulated Data)

The RNN-derived landscape closely matched the ground truth, correctly identifying the depressed attractor as deeper and more stable. This demonstrates that RNNs can serve as effective empirical discovery tools for generating testable, formal hypotheses about psychological dynamics, which can then be codified into a specific DE model for further validation.

6.2 Mode 2: DE-Informed RNN for Theory-Constrained Prediction

We compared the forecasting performance on the real-world EMA dataset. The DE-Informed RNN (Mode 2) was benchmarked against a pure data-driven GRU (Mode 1) and a theory-only Differential Equation model

Model	**Negative Affect (NA)**	**Positive Affect (PA)**	**Rumination**
Theory-Only DE	0.51 ± 0.07	0.48 ± 0.06	0.62 ± 0.09
Data-Driven GRU (Mode 1)	0.39 ± 0.05	0.41 ± 0.04	0.52 ± 0.07
DE-Informed GRU (Mode 2)	**0.35 ± 0.04**	**0.38 ± 0.04**	**0.47 ± 0.06**

Table 4: One-Step-Ahead Prediction Performance on Real-World EMA Data

The DE-Informed RNN (Mode 2) achieved superior predictive accuracy, significantly outperforming both the theoryonly and pure data-driven models (paired t-tests, $p < 0.01$). This result confirms that integrating theoretical constraints enhances generalization by preventing the RNN from overfitting to spurious correlations in the data, leading to more robust and plausible forecasts.

6.3 Mode 3: RNN as Approximator within a Hybrid Model

The hybrid model (Mode 3) provided the most nuanced insight. It successfully learned the function G(PA, Rumination), which describes the context-dependent emotional inertia. Analysis of the learned hybrid model revealed that the effective decay rate of negative affect (i.e., G(...)) was significantly lower (higher inertia) during periods of high rumination and low positive affect. Conversely, inertia was lower (faster recovery from negative affect) when positive affect was high, even if rumination was moderately present.

Simulated Intervention	**Resulting Change in Negative Affect Attractor**	**Change in Inertia (Decay Rate)**
Baseline (No Intervention)	Deep, Stable	Low (High Inertia)
20% Reduction in Rumination	Shallower, Less Stable	+15%
20% Increase in Positive Affect	Shallower, Less Stable	+22%
Combined Intervention	Very Shallow, Unstable	+41%

Table 5: In Silico Intervention Simulation using the Hybrid Model

This allows for in silico experimentation. As shown in Table 3, simulating interventions by manipulating the inputs to the hybrid model suggests that increasing positive affect might be a more potent lever for reducing emotional inertia than solely targeting rumination. This generates a specific, quantifiable, and testable hypothesis for therapeutic practice.

6.4 Identifying Early Warning Signals

Applying the trained models to longitudinal data from a single participant who experienced a depressive episode, we monitored the models' dynamical indicators. The DE-Informed RNN (Mode 2) correctly detected a significant increase in both autocorrelation (a sign of critical slowing down) and variance in the negative affect time series in the weeks preceding the clinical diagnosis (van de Leemput et al., 2014). This was visually evident as a "flickering" between states in the reconstructed phase portrait, indicating the destabilization of the healthy attractor before the transition.

7. DISCUSSION

This paper introduced and validated a novel methodological synthesis for psychological science: the fusion of theorydriven differential equations (DEs) and data-driven recurrent neural networks (RNNs). Our results demonstrate that this hybrid framework successfully bridges a critical gap in the field, leveraging the respective strengths of each approach to overcome their inherent limitations. By moving beyond the traditional dichotomy of theory versus prediction, we have established a practical pathway for building formal, testable, and idiographic models of psychological processes as they unfold in time.

7.1 Theoretical Synthesis and Empirical Validation

The core contribution of this work is the operationalization of a hybrid DE-RNN framework. Our findings confirm that the three proposed fusion modes are not merely conceptual but are empirically viable and offer distinct scientific utilities.

First, **Mode 1 (RNN for Discovery)** effectively addressed the specification problem of pure dynamical systems theory (Hamaker, 2012). The successful reconstruction of the bi-stable attractor landscape from the RNN's hidden states (Table 3) demonstrates that these "black-box" models can be "cracked open" using dynamical systems analysis. This provides a rigorous, data-driven method for generating hypotheses about the underlying structure of psychological processes, such as the number and stability of affective states, when a priori theory is insufficient. This aligns with calls for more exploratory, data-intensive approaches in psychology (Fisher et al., 2018) and provides a concrete methodology for doing so without sacrificing formal mathematical interpretation.

Second, **Mode 2 (DE-Informed RNN)** directly tackled the interpretability problem of pure machine learning. By embedding a simple theoretical DE (relating negative affect and rumination) into the RNN's loss function, we created a model that was not only more accurate (Table 4) but also more theoretically plausible. The superior predictive performance of the DE-Informed RNN suggests that theoretical constraints act as a powerful regularizer, guiding the network towards solutions that are consistent with established psychological principles, thereby enhancing generalizability. This approach, inspired by Physics-Informed Neural Networks (Raissi et al., 2019), ensures that our most powerful predictive tools remain grounded in psychological theory, transforming them from pure forecasting engines into instruments of scientific discovery.

Finally, **Mode 3 (RNN as Approximator within DE)** offered a powerful new paradigm for hybrid mechanistic modeling. The finding that the function G(PA, Rumination) encoded a context-dependent inertia provides a level of nuance that is difficult to achieve with either approach alone. It formalizes the intuitive clinical idea that the same level of rumination may have a different impact depending on the individual's concurrent positive affect. The ability to run in silico interventions (Table 5) moves computational psychology from a descriptive to a generative science, allowing researchers to simulate the dynamic consequences of therapeutic strategies before implementing them in the real world, a key ambition of process-based therapy (Hofmann et al., 2020).

7.2 Implications for Psychological Science and Clinical Practice

This research represents a concrete step towards the paradigm shift called for by Hofmann, Curtiss, & Hayes (2020) from a variable-centered to a process-based science. Our framework provides the mathematical and computational tools to model the "causal forces" that govern an individual's psychological trajectory. The detection of early warning signals (increased autocorrelation and variance) preceding a depressive transition, as predicted by the theory of critical slowing down (van de Leemput et al., 2014), exemplifies this shift. It moves the focus from static risk factors to dynamic, real-time indicators of impending change, opening the door to just-in-time adaptive interventions.

Clinically, the implications are profound. The visualized attractor landscapes and simulation results provide a tangible model for case conceptualization. A therapist is no longer limited to discussing abstract "patterns" but can, in principle, visualize a client's maladaptive attractor and simulate how specific skills (e.g., cognitive defusion, behavioral activation) might alter that landscape to foster resilience. This makes the process of therapeutic change conceptualized as a movement between attractor states (Hayes et al., 2020) more transparent, measurable, and targetable. **7.3 Limitations and Future Directions**

Despite its promise, this framework is not without limitations. Our proof-of-concept implementation used relatively simple DEs and a modest-sized RNN. The psychological theories we can formalize are currently limited in complexity. Future work must integrate more sophisticated, multi-variable theories from clinical science, which may require more advanced neural architectures and training schemes.

A second challenge is scalability and accessibility. The technical expertise required to implement this framework spanning dynamical systems theory, deep learning, and clinical science is substantial. The development of userfriendly, open-source software packages will be crucial for its widespread adoption by the psychological research community.

Furthermore, the ethical implications of such powerful idiographic models must be carefully considered. The ability to forecast a client's emotional state or risk of relapse with high accuracy raises important questions about data privacy, client consent, and clinical responsibility. These ethical challenges must be addressed in parallel with technical advancements.

Finally, future research should explore the fusion of this framework with other methodological advances, such as network modeling (Borsboom, 2017) and group-level iterative estimation (GIMME), to bridge the idiographicnomothetic divide. Applying this approach to a wider range of psychological phenomena from the dynamics of social interaction to the moment-to-moment fluctuations in attention will test its generality and further refine its methodology

8. CONCLUSION

This paper has articulated and demonstrated a transformative path for psychological science by forging a rigorous synthesis between differential equations and recurrent neural networks. We have argued that the fundamental nature of psychological phenomena as temporal, non-linear, and idiographic processes (Thelen & Smith, 1994) necessitates a move beyond static, variable-centered models. The proposed hybrid DE-RNN framework directly answers this call, providing a structured methodology to formalize theory, learn from intensive longitudinal data, and derive clinically actionable insights.

The three fusion modes discovery, theory-constrained prediction, and hybrid mechanistic modeling collectively overcome the central limitations that have hindered both pure theory-driven and pure data-driven approaches. By using RNNs to uncover attractor landscapes from data, we address the specification problem that has constrained dynamical systems theory (Hamaker, 2012). By embedding theoretical DEs as constraints within RNNs, we solve the interpretability problem of black-box machine learning, ensuring that predictive models remain psychologically plausible and generalizable (Raissi et al., 2019). The successful application to emotional inertia in depression validates this synergy, demonstrating enhanced predictive accuracy alongside a deeper, mechanistic understanding of the dynamics underpinning psychopathology (Kuppens et al., 2010).

This work represents a concrete instantiation of the process-based approach advocated by leaders in the field (Hofmann et al., 2020). It provides the mathematical and computational tools to model the very "processes of change" that are the focus of modern therapeutics, shifting the scientific gaze from what variables people have to how their psychological states unfold and transform over time. The ability to visualize attractor landscapes, simulate interventions, and detect early warning signals of critical transitions (van de Leemput et al., 2014) moves the field from a science of correlates to a generative science of processes.

In conclusion, the fusion of differential equations and recurrent neural networks is more than a mere technical advance; it is a paradigm shift. It offers a unified, scalable, and rigorous framework to finally capture the flowing, ever-changing stream of human experience. By embracing the complexity of psychological dynamics, this approach promises not only to enhance the predictive power of our models but to finally deliver a mechanistic understanding of the causal forces that shape human thought, emotion, and behavior across time. The future of psychology, as a science of processes in motion, is inextricably linked to the continued development and application of this powerful methodological synthesis.

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