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RICHNESS OF THE MATHEMATICS IN ELEMENTARY STUDENTS' GROUP WORK

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Abstract
This study aims to analyze the quality of student interactions in elementary mathematics group work based on the Richness of the Mathematics indicators from the Mathematical Quality of Instruction (MQI) framework. The research focuses on how aspects of Richness of the Mathematics are reflected in students' discussions, which indicators are most dominant, and the forms of representations, explanations, and problem-solving strategies employed. A qualitative method with a multiple-case study design supported by video analysis was applied. Data were obtained from 20 video recordings of group discussions among fifth-grade students on the topic of fractions, facilitated by five elementary school teachers. The student interactions were analyzed using six Richness of the Mathematics indicators: Linking and Connection, Explanation, Mathematical Meaning and Sense-making, Multiple Procedures or Solution Methods, Patterns and Generalization, and Mathematical Language. The findings reveal that the indicators of Linking and Connection and Explanation consistently appeared across all groups, while Mathematical Meaning and Sense-making was particularly dominant (95%). In contrast, Multiple Procedures (25%), Patterns and Generalization (35%), and Mathematical Language (15%) emerged with lower frequency. These findings suggest that group work supports students in building conceptual connections and articulating ideas through visual representations but remains limited in strategy variation, generalization, and the use of formal mathematical language.
Keyword: Richness of the Mathematics, Group Work, Mathematical Quality of Instruction (MQI), Elementary Mathematics Learning

INTRODUCTION

The characteristics of mathematics learning have unique features that distinguish it from other subjects [1]. Mathematics instruction emphasizes students' active engagement with logically and conceptually structured learning experiences to build a deep understanding of mathematical concepts [2]. Mathematical ideas can be presented in various forms, such as diagrams, written symbols, oral language, real-world contexts, or manipulatives, so that abstract concepts can be bridged through concrete objects to make them more accessible. Learning activities and tasks are also designed to enhance mathematical thinking skills and problem-solving abilities in everyday life. Thus, mathematics learning should not be limited to the delivery of procedures without understanding, which often leads students to merely memorize steps [3]. These unique characteristics require careful observation and monitoring to ensure instructional quality. Direct observation of mathematics teaching and learning plays an important role in evaluating its effectiveness [4], as instructional quality substantially influences what students are able to learn in the classroom [5]. This quality, particularly in the interactions among students, teachers, and mathematical content, is a key factor in instructional success [6]. Therefore, comprehensive observation needs to consider how mathematical concepts are delivered and understood by students, including the use of visual representations, concrete models, and problem-solving strategies. Through such observation, valuable insights can be gained to improve teaching effectiveness and student learning outcomes.

Previous studies have provided important insights into the key dimensions of instructional quality. For instance, Brunner & Star (2024) examined the quality of mathematics instruction by focusing on empirically validated effective aspects and normative dimensions [1]. Lindermayer et al. (2024) investigated whether classroom-level configurations of instructional quality could be identified by considering students' aggregated perceptions [2]. Mu et al. (2022) explored the quality of mathematics instruction in terms of teacher-regulated learning and its role in developing desired student outcomes [4]. Other studies have identified multiple dimensions of instructional quality, including classroom management, cognitive activation, and student support [5]. Other scholar, in contrast, proposed five dimensions of instructional quality: conceptual focus, cognitive demand, student focus, adaptivity, and longitudinal coherence [6]. A more specific approach to examining the quality of mathematics instruction was developed by Charalambous & Litke (2018), who employed the Mathematical Quality of Instruction (MQI) framework and also discussed aspects that MQI does not fully capture [7]. Their study reorganized a set of MQI items [8] into four main dimensions: Richness of the Mathematics, Error and Imprecision, Working with Students and Mathematics, and Common Core-Aligned Student Practices.

Characterizations of mathematics instructional quality based on interactions can also be extended to student-to-student interaction. Such interactions occur through group activities and peer feedback, including discussions and peer assessment [9]. However, research on the quality of group work in mathematics education remains general and has not fully taken into account the distinctive characteristics of mathematics. For example, Lintner et al. (2023) investigated mechanisms that support student interaction in group work using video data from classroom settings [10]. Other studies have explored the application of Universal Design for Learning (UDL) in collaborative settings [11]. Steenkamp and Brink (2024) evaluated the effectiveness of peer learning, including discussion forums, peer review, and group work [12]. Video-based studies have also been conducted to examine the impact of structuring procedures in cooperative learning on teacher–student interactions during group work [13]. Furthemore, Martín-Sómer et al. (2023) designed and validated a new questionnaire to form balanced student teams based on behavior and personality traits [14].

Although various studies have examined the quality of mathematics instruction from different perspectives—such as classroom management, cognitive activation, and student support [5] or the five dimensions of instructional quality proposed by Prediger et al. (2022)—prior research has primarily focused on teachers' perspectives and general aspects of instructional management [6]. Similarly, the MQI framework developed by Hill et al. (2008) and further refined by Charalambous and Litke (2018) introduced four important dimensions, including Richness of the Mathematics, which relates to the depth of representation and conceptual understanding in mathematics [8][7]. However, the application of these dimensions has largely centered on teachers and teacher–content or teacher–student interactions, rather than student-to-student interactions. Research on students' group work in mathematics has likewise tended to focus on classroom management strategies or general cooperative learning principles, such as UDL Qu & Cross (2024), without examining in depth how student interactions contribute to the richness of mathematical content itself [11]. Even video-based studies by Lintner et al. (2023) and Völlinger et al. (2022) have not explicitly linked these interactions to the specific dimensions of the MQI framework [10][13]. In other words, no study has explicitly mapped how student-to-student interactions in elementary mathematics group work reflect the Richness of the Mathematics indicators in MQI. Yet, this dimension is particularly valuable for demonstrating the effectiveness of group work not only from a social perspective but also in terms of the mathematical quality of the interactions.

As outlined above, research on the quality of student group work has primarily emphasized general aspects of student interaction. The present study specifically explores the quality of elementary students' group work in mathematics, focusing on the Richness of the Mathematics dimension of MQI. This dimension includes six indicators: Linking and Connection, Explanation, Mathematical Meaning and Sense-Making, Multiple Procedures or Solution Methods, Patterns and Generalization, and Mathematical Language. Concentrating on Richness of the Mathematics helps ensure alignment between the MQI framework and the context of mathematics group work. Accordingly, this study investigates how students construct various mathematical representations, explain mathematical concepts during discussion, build understanding with peers, explore multiple approaches to problem-solving, and use mathematical symbols in group discussions.

Based on this rationale, the study aims to offer a new perspective by examining the quality of student-to-student interactions in elementary mathematics group work through the lens of the MQI Richness of the Mathematics indicators. Using video-recorded data, the guiding research questions are: (1) How are the aspects of Richness of the Mathematics reflected in student-to-student interactions during elementary mathematics group work? (2) Which aspects of Richness of the Mathematics most frequently emerge in student interactions? and (3) In what forms are mathematical representations, explanations, and problem-solving strategies expressed by students during group work?

LITERATURE REVIEW

Richness of the Mathematics

The richness of mathematics emphasizes the meaning of facts and procedures (e.g., by connecting representations or providing explanations) as well as a focus on key mathematical practices (e.g., problem solving, developing generalizations, and using appropriate mathematical language) [15]. It also involves the use of multiple representations, the connections among representations, mathematical explanations and justifications, and clarity around mathematical practices such as proof and reasoning [16]. According to Charalambous and Litke (2018), Richness of the Mathematics encompasses six core abilities: Linking and Connection, Explanation, Mathematical Meaning and Sense-Making, Multiple Procedures or Solution Methods, Patterns and Generalization, and Mathematical Language [7] .

The Linking and Connection aspect focuses on the exploration of explicit relationships among different representations of mathematical concepts [17]. The Explanation aspect highlights the ways in which mathematical concepts are clearly and meaningfully communicated [18]. Mathematical Meaning and Sense-Making directs attention to the extent to which instruction supports students in constructing meaning and understanding of the mathematical concepts being studied [19]. The Multiple Procedures or Solution Methods aspect evaluates how teachers and students use and discuss diverse approaches to solving a mathematical problem [20]. The Patterns and Generalization aspect examines how teachers and students recognize mathematical patterns and use them to develop generalizations [21]. Finally, the Mathematical Language aspect assesses the depth and density of the use of mathematical terms and symbols during the learning process [22]. Mathematics learning that is rich in content involves the use of multiple tools, including representational tools (symbol systems, graphic organizers, diagrams, illustrations, tables), physical tools (models, measuring instruments), and digital tools (computers, software, the internet) [23]. Such learning can also be diversified by applying mathematical concepts across a variety of contexts drawn from real-life situations. It encourages students to develop critical thinking skills—not only to know and use methods but also to evaluate the results obtained and to reflect on whether the application of mathematical concepts is appropriate. Rich mathematical learning is further characterized by opportunities for students to test mathematical concepts through discussion, in which they defend and justify their solutions.

This dimension also considers whether teachers and students use more than one model to represent mathematical content. Multiple models may include, for example, graphs, equations, and tables, or drawings, numerical procedures, and narratives. For instance, if a problem is presented symbolically or through a concrete scenario but is interpreted and solved only with a graphical model, this does not count as multiple-model use. However, if the concrete scenario is used to understand and manipulate a graphical model, this is considered as engaging multiple models [24]. Rich mathematics instruction does not merely provide routine exercises to build procedural fluency but also includes more open-ended problem-solving tasks that promote exploration, creativity, and mathematically rich thinking [25]. Moreover, the context of mathematics learning can be adapted to students' interests, which are themselves enriched by their everyday life experiences [26]

Student Interaction in Mathematics Learning

Student interaction refers to the reciprocal processes among students in a comprehensive manner that encompasses important domains of diversity, without imposing limitations on its interpretation or application [27]. Johnson and Johnson (2008) argue that social interaction among students fosters interdependence among them and mediates the construction and understanding of knowledge in cooperative learning. Student interaction also encourages active engagement in providing feedback during the learning process [29]. Swan (2002) found that peer interaction is associated with both knowledge acquisition and students' perceived satisfaction in learning [30]. Swan's (2002) findings also indicate that peer interaction centers on the development of social presence, which helps reduce psychological distance among students [30].

Researchers have highlighted that learning through peer interaction generates more positive outcomes compared to learning in isolation [31]. Students gain opportunities to collaborate and discuss content with their peers [32]. Their reasoning abilities also improve, as peer group interactions encourage them to explain concepts, compare strategies, and draw conclusions [33]. Beyond fostering reasoning, such interactions help lower students' affective barriers [34]. Studies also reveal that peer interaction fosters positive interdependence during learning activities [35]. Additionally, students develop more positive attitudes toward academic topics, instructors, and their own self-efficacy as learners, while becoming more tolerant of peers who differ from them [36]. Collaborative learning also enables students to develop communication, leadership, and conflict-resolution skills [31]. According to Johnson, five elements are essential for successful student interaction: positive interdependence, individual accountability, promotive interaction, social skills, and group processing [37]. Furthermore, researchers have found that peer interaction in cooperative learning supports students' self-regulation as well as their social and emotional learning skills [38].

The quality of student interaction is often assessed through group work in learning settings. Previous studies have examined group work effectiveness by measuring the extent of student participation in discussions, such as the number of students who actively speak [39]. Qu & Cross (2024), however, investigated what makes group work effective using the principles of Universal Design for Learning (UDL) [11]. These principles include designing tasks that engage all students in the completion process; providing multiple means of action and expression, such as assessment options and group feedback; and offering multiple representations of learning materials to support understanding and communication [11]. Other studies have evaluated the effectiveness of peer learning through discussion forums, peer reviews, and group work [12]. In discussion forums, for example, researchers examined how students responded to a given question and how at least two peers engaged with those responses. Peer review involved students providing feedback on their classmates' draft assignments. Meanwhile, group work required students to collaborate to complete assigned tasks. Vanessa (2022) conducted a video-based study analyzing how students worked on case-based tasks by observing their active participation in group discussions, their use of conceptual knowledge during discussions, and their transactional communication during concept-based conversations [13].

Johnson and Johnson (2015) proposed four perspectives for understanding cooperative learning: cognitive-developmental, social-cognitive, behavioral learning theory, and social interdependence. They also outlined five critical elements of group learning [35]. The first is positive interdependence, in which individual success is tied to the success of the group, meaning that a group cannot succeed unless each member succeeds. The second is promotive face-to-face interaction, where members assist one another in completing tasks. The third is individual accountability, which requires members to recognize their responsibility to contribute to group activities so that collective goals can be achieved. The fourth is social skills, where each member must consciously develop interpersonal and group skills necessary for effective collaboration. The final element is group processing, where members reflect collectively to identify which behaviors facilitated or hindered success and what areas require improvement. Similarly, Slavin (2014) suggests that group success can be achieved when motivation, social cohesion, cognition, and development complement one another [41].

METHOD

The focus of this study was to analyze the aspects of Richness of the Mathematics reflected in peer interactions during group work in elementary mathematics learning. Specifically, it examined which aspects of Richness of the Mathematics were most dominant in student interactions, and how students used representations of concepts, explanations, and problem-solving strategies during their interactions. This required an in-depth, detailed, and careful exploration of teachers and students in classroom learning, particularly with regard to student-to-student interaction. The data collected were identified, analyzed, and qualitatively described based on actual conditions in the field. Accordingly, this study employed a qualitative approach with an exploratory descriptive design [42] using a multiple case study framework supported by video analysis.

The study observed 20 video recordings of students' mathematics group work facilitated by five elementary school teachers in Grade 5 classrooms in an Indonesian city. The participating teachers were certified educators, had a minimum of five years of teaching experience, taught upper-grade classes, and designed lesson plans that incorporated mathematics group work. Each teacher guided several groups consisting of four to five students. The lessons focused on fractions, including fraction concepts, fraction comparison, and fraction operations.

We recorded students' discussions and interactions as they completed teacher-assigned group tasks, with an average duration of 40 minutes. Cameras were placed on each group's table, as well as in general classroom positions to capture overall activities. In addition, student worksheets on fractions were collected to support the interpretation of students' answers. Student conversations were transcribed and aligned with their written work or visual representations.

The analytical instrument was based on the Richness of the Mathematics indicators from the Mathematical Quality of Instruction (MQI) framework, applied to student interactions in group work. The indicators and their codes are presented below.

Table 1. Indicators of Richness of the Mathematics from MQI

Indicator	Descriptor	Code
Linking and Connection	Peer interactions connect fraction concepts with real-life contexts	RM1
Explanation	Explanations of fraction content during peer interaction correctly employ mathematical tools (graphs, diagrams, number lines, tables, pictures)	RM2
Mathematical Meaning and Sense-making	Peer interactions construct the meaning of ideas related to fraction concepts	RM3
Multiple Procedures or Solution Method	Peer interactions use and discuss multiple approaches to solve fraction-related problems	RM4
Patterns and Generalization	Peer interactions identify numerical patterns and apply them to generalize fraction concepts	RM5
Mathematical Language	Density of mathematical language used during peer interaction	RM6

Each video was independently observed by the researchers using the Richness of the Mathematics indicators. Relevant data were screened, selected, summarized, and grouped according to the indicator codes. The coded results were summarized per group, then aggregated per teacher to identify general patterns. Frequencies of indicator occurrences were calculated to determine the most dominant aspects. To enrich the analysis of the Richness of the Mathematics, excerpts from student conversations and samples of their work were selected to illustrate each indicator. The validity of the data was ensured through source triangulation. Data were drawn from video recordings, student conversation transcripts, and student worksheets, thus providing a more comprehensive interpretation that did not rely solely on a single type of data.

RESULTS

The analysis of video recordings revealed that not all aspects of Richness of the Mathematics appeared equally in students' interactions during group work. Some aspects were consistently present across nearly all groups, while others were rarely observed. Among the six aspects of Richness of the Mathematics, the most frequently identified were Linking and Connection and Explanation. The variation in the occurrence of these aspects was influenced by both the dynamics of student discussions and the ways in which teachers facilitated group work.

The aspect of Linking and Connection was particularly dominant in group discussions. Students actively connected fraction concepts with relevant mathematical procedures. For instance, nearly all groups solved fraction addition tasks by finding a common denominator through the Least Common Multiple (LCM). The following excerpt illustrates how students collaboratively built connections between fraction concepts and solution procedures:

Student 1: “For problem number 3, the denominators are different. So, how do we add them?”
Student 2: “We need to make the denominators the same.”
Student 3: “Yes, they have to be the same. The denominators are 3, 4, and 12.”
Student 1: “So, 3, 6, 9, 12… then…”
Student 3: “This one… 4, 8, 12… and the other is 12, 24.”
Student 1: “So the denominator becomes 12.”
Student 3: “Yes.”

The group then connected this reasoning with the concept of equivalent fractions for the numbers being added. This dialogue demonstrates that students were not merely recalling procedures, but were gradually deriving the steps for finding a common denominator by comparing multiples of different numbers. The process was then linked to equivalent fractions, illustrating how Linking and Connection naturally emerged in group discussions. In addition to the dialogue, students also demonstrated their interconnected thinking in their written work. As shown in Figure 1, students equalized the denominators of 3, 4, and 12 by identifying common multiples and then wrote equivalent fractions to facilitate addition.

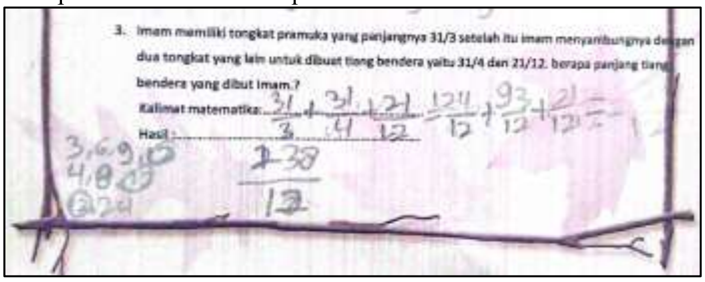

Figure 1. Students’ work on finding common denominator by identifying common multiples.

In the concept of comparing fractions, students connected it with the concepts of “greater than” or “less than” numbers. They constructed number lines to arrange fractions either from the smallest to the largest or vice versa. In addition, students also related the concept of fractions to everyday life contexts. This was evident when they worked on word problems, for example by imagining the division of food or other concrete objects. This indicates that group interaction facilitated the process of linking mathematical ideas with both formal procedures and real-life experiences.

The Explanation aspect also appeared strongly in students’ group discussions. Students explained the concept of fractions through visual representations, such as drawing squares divided into equal parts, shading to show the numerator, or using number lines. These explanations often emerged in the form of peer corrections, for instance when a group member shaded the square incorrectly or misstated the solution steps. This could be seen in students’ comments to their peers such as, “the square must be divided into 6 equal parts”; “since the numerator is 3, then 3 parts should be shaded.” Students used circles, squares, and number lines to represent the concept of fractions. This can be seen in Figure 2.

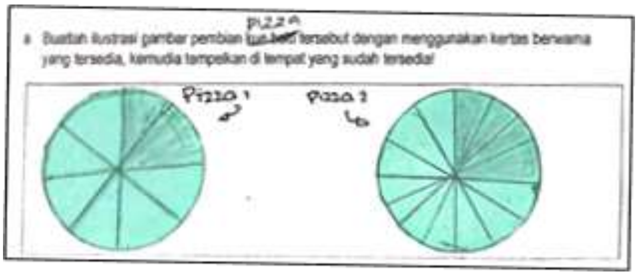

Figure 2. Students’ work on presenting the concept of fractions.

Thus, Explanation played an important role in maintaining the clarity of procedures and fostering shared understanding within the group, although there were still some groups whose representations did not fully match the given problems.

The aspect of Mathematical Meaning and Sense-making appeared with moderate intensity in students’ interactions. Several groups attempted to make sense of fractions by linking symbols to concrete and visual representations, for example stating that “two-fourths is equal to one-half because two parts out of four is one-half.” However, there were still groups whose interactions did not result in accurate meaning-making. For instance, a group tried to understand the meaning of $\frac{2}{3}$ by drawing a square divided into three unequal parts and then shading two of them, as shown in Figure 3 below.

Figure 3. Students' work on dividing a square into three unequal parts

This indicates that the groups did not only emphasize the procedures but also tried to grasp the conceptual meaning of fractions. Similarly, for the concept of comparing fractions, the groups attempted to compare the areas of divided squares to determine which fraction was the smallest or largest.

The aspect of Multiple Procedures or Solution Methods was hardly visible across the groups. Students tended to rely on a single solution method that was considered the easiest. For fraction addition problems, groups simply equalized the denominators and then added the fractions. They did not provide alternative strategies, such as using number line representations or comparing two different approaches. For fraction comparison problems, groups employed two strategies: comparing the areas of shaded parts and ordering fractions on a number line. However, these strategies were followed strictly according to the steps outlined in the student worksheets designed by the teacher. This indicates that the variation in problem-solving strategies emerging in group discussions largely depended on the teacher's instructional design.

The aspect of Patterns and Generalization was also rarely observed. Group interactions related to this aspect occurred only in recognizing numerical multiples to equalize denominators. The groups had not yet attempted to identify broader numerical patterns or generalize procedures to other problems. Instead, they placed greater emphasis on solving the given problems rather than exploring wider mathematical regularities. These findings suggest that group work placed more focus on procedural completion than on pattern exploration.

Regarding the aspect of Mathematical Language, groups used mathematical terms only in a relatively simple way. Group members employed fraction-related terms such as numerator, denominator, "make them the same first," or "greater than/less than" in their discussions. However, the density of mathematical language used was relatively limited. Many students tended to rely more on everyday language rather than formal terminology, which suggests that mathematical interaction was not yet fully established.

The frequency of the occurrence of the Richness of the Mathematics indicators is presented in Figure 4. These findings address the second research question of this study.

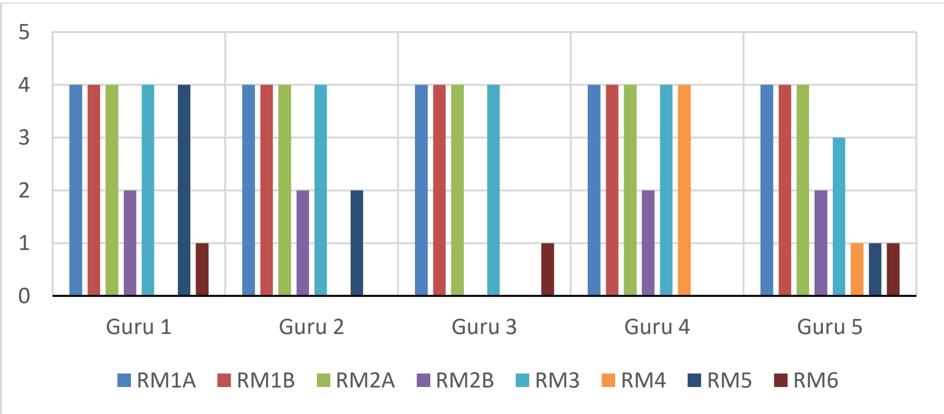

Figure 4. Frequency of the occurrence of Richness of the Mathematics indicators

Figure 4 shows that across the instruction of the five teachers, the indicators RM1A, RM1B, and RM2A consistently appeared. This indicates that in all classrooms, students engaged in interactions to connect mathematical ideas and to provide procedural explanations to their peers. When examined per teacher, some variation is evident in the occurrence of other indicators. In Teacher 1's instruction, all groups demonstrated interactions in all aspects except RM4 (Multiple Procedures or Solution Methods), which did not appear at all. Teacher 2 displayed a similar pattern to Teacher 1, with RM5 (Patterns and Generalization) and RM6 (Mathematical Language) emerging only in some groups. In Teacher 3's instruction, RM2B did not appear, while RM5 and RM6 were also very limited. Interestingly, in Teacher 4's instruction, RM4 (Multiple Procedures or Solution Methods) appeared in all groups, although RM5 and RM6 were completely absent. This highlights that teacher guidance can open opportunities for students to explore more than one solution method. Meanwhile, in Teacher 5's instruction, almost all aspects appeared, though not evenly. All groups demonstrated RM1A, RM1B, RM2A, and RM3 (Mathematical Meaning and Sense-making), but RM4, RM5, and RM6 appeared in only one group. Thus, while a common dominant pattern of RM1 and RM2A was evident across all teachers, there were variations in the occurrence of RM3 through RM6. This suggests that the characteristics

of each teacher's instruction influenced the types of mathematical interactions that emerged within student groups.
Based on the percentage of the occurrence of the Richness of the Mathematics indicators across all student groups, the results are presented in Table 2 below.

Table 2. Percentage of the occurrence of Richness of the Mathematics indicators across groups

Indicator	Code	Number of Groups (20)	Percentage (%)
Linking and Connection	RM1A	20	100 %
	RM1B	20	100%
Explanation	RM2A	20	100%
	RM2B	8	40%
Mathematical Meaning and Sense-making	RM3	19	95%
Multiple Procedures or Solution Method	RM4	5	25%
Patterns and Generalization	RM5	7	35%
Mathematical Language	RM6	3	15%

Table 2 shows that the indicators Linking and Connection (RM1A and RM1B) as well as Explanation (RM2A) appeared in all groups. In contrast, Explanation (RM2B) appeared in only eight groups (40%), indicating a limitation in providing deeper or more varied explanations during student interactions. Furthermore, Mathematical Meaning and Sense-making (RM3) appeared quite dominantly, with a frequency of 95% (19 groups). However, indicators that demand more complex strategies, such as Multiple Procedures or Solution Methods (RM4), were found in only five groups (25%), and Patterns and Generalization (RM5) in seven groups (35%). Mathematical Language (RM6) was the least frequently observed aspect, appearing in only three groups (15%). Overall, these findings indicate that more basic aspects such as conceptual connections, simple explanations, and mathematical sense-making were relatively well practiced in student interactions. However, higher-level abilities such as using multiple solution strategies, recognizing generalizable patterns, and employing formal mathematical language still need to be strengthened in group learning processes.
The forms of conceptual representation used by the students included efforts to connect the concept of fractions with related concepts such as multiplication, least common multiples, and comparative relations of "greater than" or "less than." Students also linked the concept of fractions to real-life contexts. These representations were reinforced by the explanations students provided, for example through visualizing fractions using concrete manipulatives, drawings of geometric shapes, and number lines. Such explanations demonstrate how students constructed meaning of the concepts of fractions, comparisons, and fraction addition operations. The problem-solving strategies employed by students tended to be singular, relying primarily on visual representations, although some groups also used number lines. The numerical patterns students used were mainly patterns of whole number multiples. The mathematical language most frequently employed included terms such as numerator, denominator, and comparison of fractions.

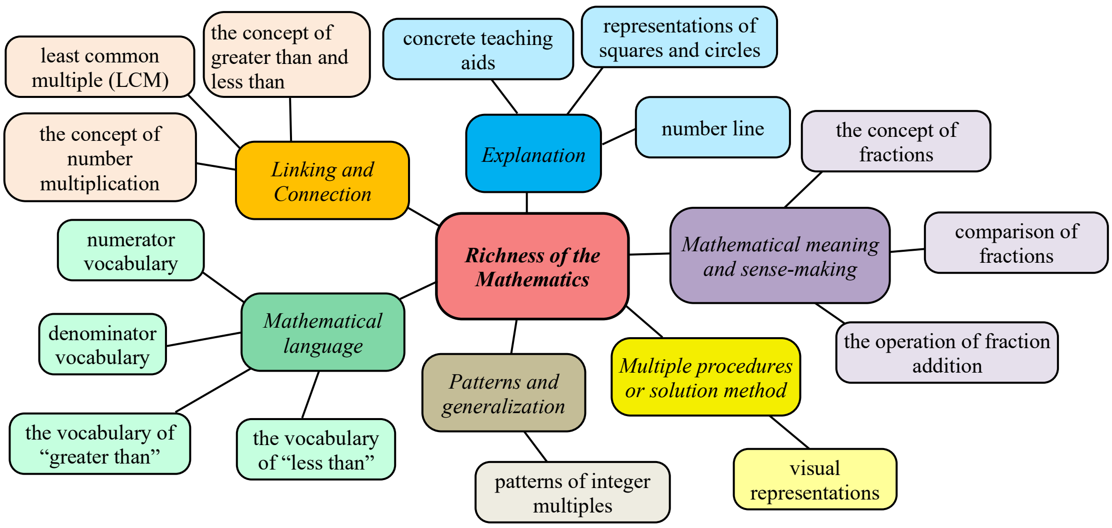

Figure 5. Representations of the emerging Richness of the Mathematics indicators

DISCUSSION

The frequency of the indicators of Richness of the Mathematics in student interactions during group work varied considerably. The frequent occurrence of the indicators Linking and Connection and Explanation suggests that group work encouraged students to share reasoning and build collective understanding. This finding supports the work of Volker Schoer and Jose G. Clavel, who showed that collaborative interaction can enhance students' ability to articulate their thinking and connect mathematical ideas [43]. It also highlights that group discussions provide space for students to establish connections between concepts and to minimize misconceptions when translating word problems into mathematical models (such as drawings, number lines, and fractions) [44]. In other words, group work effectively facilitates students' conceptual understanding, as also confirmed by [17][19].

The Explanation indicator was evident in the drawings produced by students with the support of concrete media. Such media enriched students' explanations and mathematical connections in understanding the concepts [45]. Although the quality of explanations varied—since not all students were able to fully articulate mathematical ideas in writing [46], represent mathematical concepts in a well-structured manner [47], or interpret data from diagrams [48]. However, this finding aligns with the study by Fadzil et al. (2021), which showed that group discussions encourage students to provide explanations of their ideas [49]. Teachers can facilitate this process by guiding discussions so that each group member systematically explains their visualizations.

The next most frequent indicator was Mathematical Meaning and Sense-making. This demonstrates that some student interactions in groups attempted to understand the conceptual meaning of fractions, rather than merely following mechanical procedures. Student interactions at this stage built shared conceptual meaning as a precondition for deeper understanding [50]. Efforts to connect symbols with concrete representations, for instance stating that "two-fourths is the same as one-half because two parts out of four equal one-half," reflect symbolic–conceptual understanding. However, some group interactions still revealed misunderstandings. This is consistent with Zhao et al. (2021), who emphasized that sense-making in mathematics requires teacher scaffolding [20]. Thus, during group work, teacher intervention is crucial to promote deeper reasoning beyond procedural execution [51].

In contrast, the indicators Multiple Procedures or Solution Methods and Patterns and Generalization appeared relatively infrequently. Student interactions tended to be fixed on a single procedure (e.g., finding common denominators) without exploring alternative strategies. Yet the ability to identify patterns and generalize is essential in mathematics [52]. This contrasts with Grobe's (2022) study, which reported that collaboration can help students generate more solution methods, though its effectiveness depends on the task and the structure of the group work [54]. The limited exploration of alternative strategies or mathematical generalization in this study suggests that students remained reliant on the single procedures taught by the teacher. Mainali's (2021) findings confirm that many students employ only one (often procedural) method, influenced by teaching practices, assessments, and attitudes toward cognitive risk-taking [55]. Therefore, teachers should design more open-ended tasks to stimulate creativity and mathematical generalization [56]. Teachers can also respond to student discussions with questions, comments, or prompts that encourage reflection and exploration of other strategies [57].

The least frequent indicator was Mathematical Language. Formal mathematical language was rarely used by students during their group interactions. This finding reinforces the study by Larsson et al. (2024), which showed that in group discussions, some students simply agreed with peers' answers without providing arguments or evaluations verbally [58]. Students often mixed everyday language with formal mathematical terms, and the density of mathematical language use during discussions remained relatively low. Yet, as Nick Otuma (2023) argued, frequent use of formal terminology is essential for building long-term mathematical competence [22]. This contrasts with Nusantara (2025), who found that group discussions can enhance communication quality as students co-construct meaning through mathematical conversations [59]. These findings collectively underscore the importance of teacher guidance in enriching students' mathematical vocabulary during group discussions [60].

Overall, the findings of this study affirm that group work has the potential to enrich students' mathematical understanding, particularly in terms of explanations and conceptual connections. However, teacher intervention is still necessary to stimulate exploration of alternative procedures and generalization. Teachers can employ open-ended questions, problem-solving challenges that allow multiple approaches, and structured group reflection to strengthen underrepresented aspects, making learning richer in mathematical content.

This study also maps specifically how the indicators of Richness of the Mathematics are reflected in student-to-student interactions (rather than only teacher–student interactions). This complements prior studies (Hill et al., 2008; Charalambous & Litke, 2018), which mainly focused on instructional quality from the teacher's perspective. Thus, this study broadens our understanding of how student group work can support the quality of mathematics learning at the elementary level.

CONCLUSION

This study shows that the quality of student interaction in group work during elementary mathematics learning can be mapped through the Richness of the Mathematics indicators of the MQI framework. The video analysis revealed that the indicators of Linking and Connection and Explanation were the most dominant aspects emerging in group discussions. This suggests that group work encourages students to connect mathematical ideas and explain problem-solving procedures both visually and verbally. In addition, the indicator of Mathematical Meaning and Sense-Making also appeared quite frequently, indicating students' efforts to build conceptual understanding from symbols and concrete representations. However, indicators that demand more complex mathematical abilities, such as Multiple Procedures or Solution Method, Patterns and Generalization, and Mathematical Language, were still rarely observed. Students tended to rely on a single procedure taught by the teacher, explored fewer alternative strategies, and often used everyday language instead of formal mathematical terminology.

Thus, it can be concluded that group work has the potential to enrich students' mathematical understanding, particularly in the aspects of conceptual connections and explanation. Nevertheless, teacher support is still needed in designing more open-ended tasks that stimulate the exploration of various strategies, encourage pattern generalization, and strengthen the use of formal mathematical language in group discussions. These efforts are crucial so that students' collaborative interaction not only enhances conceptual understanding but also broadens higher-order mathematical thinking skills.

IMPLICATION OF THE STUDY

The findings of this study provide important implications for mathematics teaching practices. First, teachers need to design group tasks that not only emphasize procedural completion but also require the exploration of multiple strategies and mathematical patterns. Open-ended tasks will give students opportunities to develop creativity, make generalizations, and use mathematical language more richly. Second, teachers can provide scaffolding in the form of open-ended questions, prompts to compare different methods, and emphasis on using formal terminology during group discussions. This aligns with efforts to increase the density of mathematical language and foster students' long-term competencies. Third, this study underscores the importance of expanding the use of the MQI framework, particularly the Richness of the Mathematics dimension, not only in teacher–student interactions but also in student–student interactions within group work. These implications can serve as a foundation for developing cooperative learning strategies that emphasize the mathematical quality of interaction, while also offering new directions for future research to investigate the relationship between group interaction quality and student learning outcomes at the elementary level.

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