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COMPARISON OF EFFECTIVE GREEN´S FUNCTION IN EQUIVALENT CIRCUITS ASSOCIATED WITH BASIC ELECTROCHEMICAL CELLS AND LI-ION BATTERIES

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ABSTRACT:
In this study, a novel approach to characterize and model electrochemical cell discharge, focused specifically on lithium batteries, is presented. The download process is analyzed in detail using Green's functions to introduce the electromagnetic inertia breaking (EIB). This approach allows a deeper understanding of the underlying mechanisms. Both theoretical and practical aspects are examined, highlighting the importance of this methodology in improving the performance and efficiency of lithium batteries. The results obtained offer a promising perspective for the design and optimization of advanced electrochemical systems.

KEYWORDS: Green´s functions, lithium-ion batteries, equivalent RC circuits, Electromagnetic Inertia Breaking (EIB)

1) INTRODUCTION:

In the constant use of rechargeable batteries in all areas and everyday contexts, they require a recurring connection to power supplies [1-2]. Each charge needs an initial electrical impulse to break the electrical inertia of the battery [3-4], It is, important to take into account the overload of installations that have a network of multiple connections and use a greater amount of energy [5-6].

This work aims at characterizing this initial impulse of electromagnetic inertia break (EIB) that is always present. The Green´s function in the model for batteries called RC Simple Circuit [7] are used in the use of electrochemical and lithium batteries [8-9].

As rechargeable batteries become ubiquitous, understanding the dynamics of their operation is crucial for improving their efficiency and reliability [10-12]. This is particularly important in systems where multiple batteries are used simultaneously [13], as the cumulative effects of the EIB can lead to significant energy inefficiencies and strain on power infrastructure. Addressing this issue through mathematical modeling and practical experimentation can pave the way for innovations in energy management systems, enabling the development of more sustainable technologies [5, 14].

The implications of such research extend beyond immediate applications. By comprehensively analyzing the EIB phenomenon, researchers can gain insights into the broader principles governing energy transfer and storage, contributing to advancements in related fields such as renewable energy integration and electric grid optimization [15-16].

Lithium-ion batteries have become integral to the advancement of modern technology, powering a vast array of devices from portable electronics like smartphones and laptops to larger applications such as electric vehicles and renewable energy storage systems [17-18]. The efficiency, longevity, and energy density of lithium-ion batteries have made them the preferred choice in many industries [19]. However, despite their widespread adoption, the underlying mechanisms that govern their operation are complex and not fully understood, presenting a rich field of study for researchers and scientists [12].

The core component of a lithium-ion battery is the electrochemical cell, which consists of an anode, a cathode, and an electrolyte. During the charging and discharging processes, lithium ions move between the anode and cathode through the electrolyte, while electrons travel through an external circuit, providing electrical energy [12]. This seemingly simple mechanism is governed by intricate electrochemical reactions and transport phenomena that occur at the molecular and atomic levels [20-21].

To understand these processes in detail, it is ssential to delve into the principles of electrochemistry that dictate the behavior of these cells [22]. Electrochemical analysis provides a framework for examining the reactions at the electrodes, the movement of ions through the electrolyte, and the overall cell performance. By breaking down these components, we can begin to model the functioning of the entire battery system [23].

In addition to their practical applications, lithium-ion batteries serve as a model system for exploring fundamental questions in electrochemistry and materials science. By investigating their behavior under various operating conditions, researchers can uncover principles that may be applicable to other energy storage technologies, such as solid-state batteries and fuel cells [5-6, 24]. This broader perspective highlights the importance of studying these systems not only for their immediate utility but also for their potential to inspire new innovations in energy science.

This article aims to bridge the gap between the abstract mathematical concepts and their practical applications in electrochemistry, making the topic accessible to scientists and researchers from various disciplines [25-26]. This work will begin by exploring the basic principles of electrochemical cells, providing a foundation for understanding the subsequent discussions. Next, the Green's functions will be introduced, elucidating their mathematical formulation and significance [27]. Finally, how Green's functions can be applied to model the processes within lithium-ion batteries will be demonstrated, offering insights into their efficiency and potential areas for enhancement.

Through this integrative approach, it is expected to provide readers with a deeper appreciation of the complexities involved in lithium-ion battery technology and the innovative methods used to analyze and optimize these critical energy storage devices. This article aims to offer valuable perspective on the world of lithium-ion batteries to chemists, physicists, engineers, and scientists from other fields [2, 7].

2) METHODS AND METHODOLOGY:

The equivalent circuit model shall be used simultaneously for ex-periments with simple electrochemical cells, such as for lithium bat-teries. For both cases, our purpose is to determine, through Green's function, the constant of inertia breaking from such sources, which will be very important for the prediction of overloads on power lines.

The experimental device developed is an electrolytic cell. It was built using an electrolyte made of 7, 8 and 11% sulfuric acid solution in a 400 ml beaker at 25° C, as illustrated in Figure 1. Two rectangular copper and aluminum metal plates of dimensions 10x2x0.1 cm were used as electrodes. The electrodes were submerged 600 seconds, during which voltage and resistance were measured at certain time intervals.

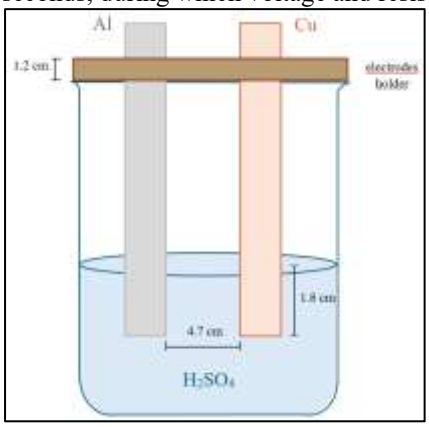

Figure 1. Electrochemical cell

Three cell phone lithium-ion batteries were used with the following characteristics:

- Battery 1: 2200 mAh, 8.38Wh Limited charge voltage: 4.35 V
- Battery 2: 3000 mAh, 11.4Wh Limited charge voltage: 4.4 V
- Battery 3: 2000 mAh, 7.6Wh Limited charge voltage: 4.35 V

To measure the potential difference of the batteries, the BioPac Student Lab device with MP44 system was used, especially for measuring human electrical signals. The circuit was constructed to decrease the voltage given by the battery using a voltage divider circuit, and observe its behavior, using a conversion factor to the nominal reference voltage of the battery. The experimental device is shown in Figure 2.

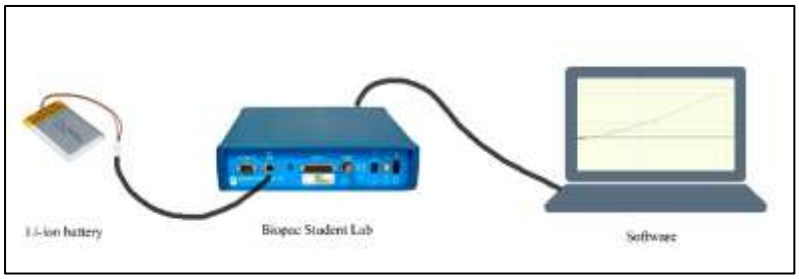

Figure 2. Experimental dispositive using Biopac Student Lab Analysis

To determine the value of the electrical inertia breakdown, an exponential regression analysis adjustment was pe

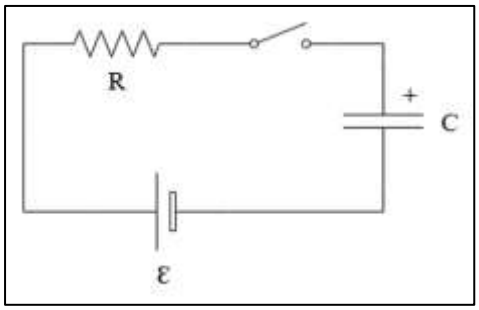

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Figure 3. RC Circuit

$$V(t) = Ae^{kt}$$

The value of k determines the approximation of the value $\tau = \frac{1}{RC}$, which gives the order of $\Delta\tau$ in terms of seconds. Suppose the following the ordinary DE needs to be solved

$$\frac{dq}{dt} = -\frac{q(t)}{RC} + \frac{V(t)}{R}$$

which may represent the charge of a RC circuit (Figure 3) with resistance R and capacitance C as a function of time by the Kirchoff voltage law under the influence of an external impulse $V(t)$. Let us first consider the special problem that occurs when the ions are in rest up to the time $t = \tau$ and then set in motion by a sudden "voltage". This implies that the external impulse $V(t)$ exists only during a small interval, say from τ to $\tau + \Delta\tau$

$$\frac{dq}{dt} = -\frac{q(t)}{RC} \qquad (t > \tau + \Delta\tau)$$

which has the solution

$$q(t) = Ae^{-\frac{t}{RC}} \qquad (t > \tau + \Delta\tau).$$

Then $Ae^{-\frac{\tau}{RC}} = I$, which yields the solution

$$q(t) = \begin{cases} 0 & t < \tau \\ Ie^{-(1/RC)(t-\tau)} & t > \tau \end{cases}$$

Finally, suppose that a continuous voltage has been acting on the particle. A voltage $V(t)$ acting at time τ would produce in the interval $d\tau$ an impulse

$$dI = V(t)\, d\tau$$

which would resemble a voltage because of its short duration. The continuous action of the voltage should then have the cumulative effect on countless impulses dI delivered to the charge. Then

$$q(t) = \int_{\tau_0}^{t} \frac{V(\tau)d\tau}{R} e^{-(t-\tau)/RC} \quad t > \tau_0,$$

assuming $q(t) = 0$ and $V(t) = 0$ prior to τ_0, this is

$$q(t) = \begin{cases} 0 & t < \tau_0 \\ \int_{\tau_0}^{t} \frac{V(\tau)d\tau}{R} e^{-(t-\tau)/RC} & t > \tau_0 \end{cases}$$

or, alternatively,

$$q(t) = \int_{-\infty}^{t} G(t,\tau)V(\tau)d\tau \quad \text{all } t$$

where

$$G(t,\tau) = \begin{cases} 0 & t < \tau, \\ \frac{1}{R} e^{-(t-\tau)/RC} & t > \tau. \end{cases}$$

The function $G(t,\tau)$ represents physically the response at time t to a unit impulse delivered at time τ. It is known as the influence function or as Green's function.

The voltage across the resistor, V_{Biopac}, is found by Ohm's Law:

$$V_{Biopac}(t) = R\frac{dq}{dt}$$

Substitution of $q(t)$ into V_{Biopac} (see appendix A) gives an expression for the output voltage

$$V_{Biopac}(t) = \int_{0}^{t} e^{-(t-\tau)/RC} \frac{dV(\tau)}{d\tau} d\tau = \int_{-\infty}^{t} \widehat{G}(t,\tau)\, V(t)\, d\tau$$

where $\widehat{G}(t,\tau) = e^{-(t-\tau)/RC}\frac{d}{d\tau}$ is an operator.

3) Results

Measurements in experimental devices yielded the following results in Table 1. Each of the cells and batteries present a resistance and rupture of measurable electro-magnetic inertia, which can determine the value of capacitance, that is why it is a dependent variable.

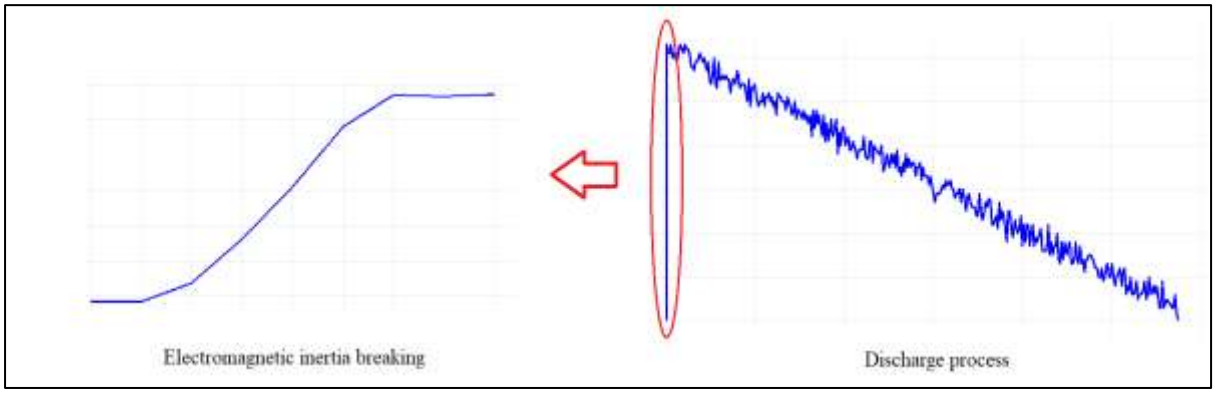


Figure 4. EIB shape in discharge process of batteries

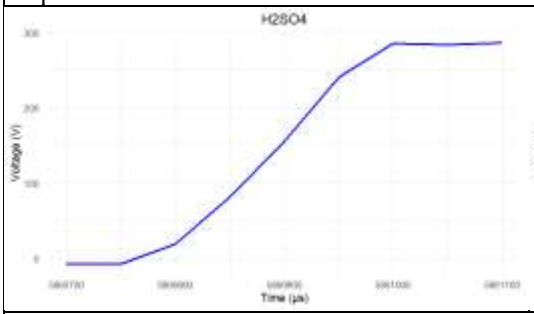
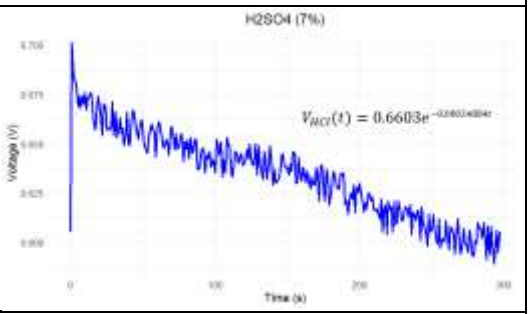
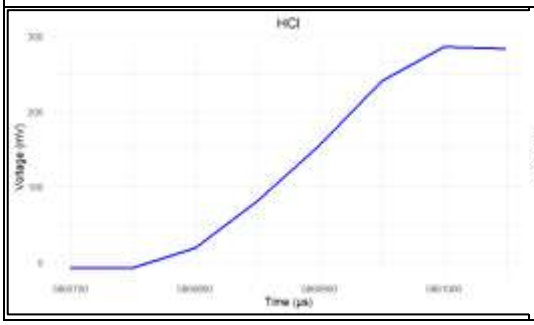
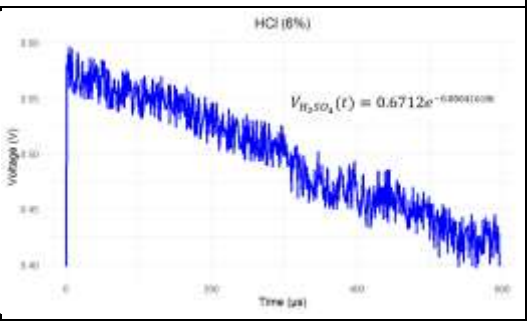
Below are the electromagnetic inertia break charts along with the discharge process of each device. Observe how the break is sudden, reaches a maximum potential difference value and then decreases at an exponential negative rate, which will allow you to estimate your values with exponential regression adjustments.

Table 1. Exponential Regression Equations

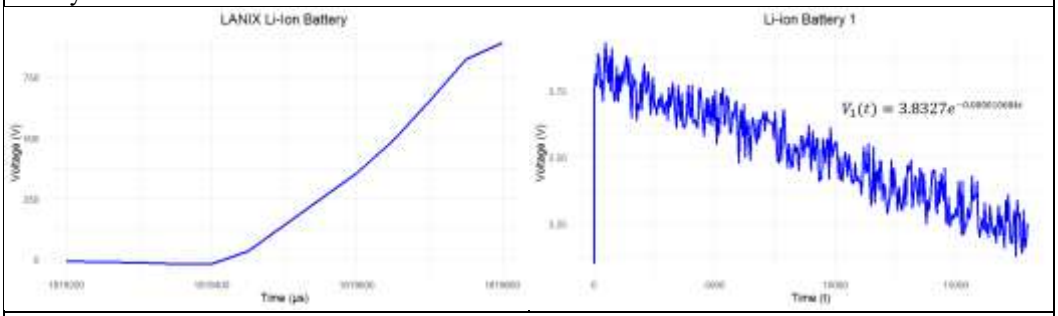
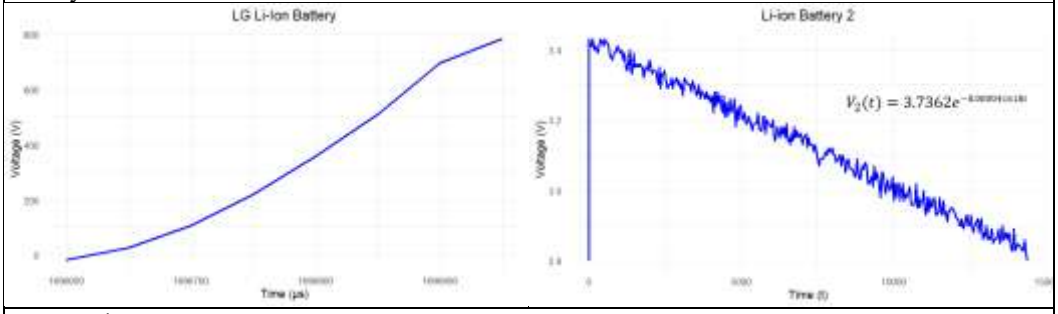
Case	**Equation**
HCl	$V_{HCl}(t) = 0.6603e^{-0.00024884t}$
H_2SO_4	$V_{H_2SO_4}(t) = 0.6712e^{-0.00041618t}$
Battery 1	$V_1(t) = 3.8327e^{-0.000010684t}$

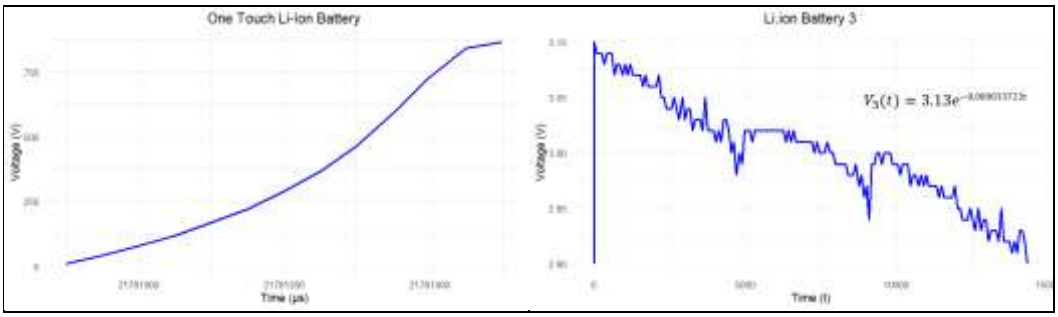
Battery 2	$V_2(t) = 3.7362e^{-0.000041618t}$
Battery 3	$V_3(t) = 3.13e^{-0.000033723t}$

Table 2. Behavior of device downloads

Electromagnetic Inertia Breaking	**Voltage Download**
Sulphuric Acid	
	
Clorhydric Acid	
	

For lithium-ion batteries, the following results were obtained as described above. The measurement scale of time is given in terms of "t", a time interval defined as t=1.31 s, t=1.336 and t=1.998 for batteries 1, 2 and 3 respectively.

Battery 1

Battery 2

Battery 3[1]



[1] Not considered due to conversion factor.

Table 3. Electromagnetic inertia breaking in tests

Cell	R (MΩ)	τ (s)	C(α) (mF)
Sulphuric Acid 1	1.4	0.000188967	3.78
Sulphuric Acid 2	1.1	0.000128015	7.11
Clorhydric Acid 1	1.9	0.000248837	2.115
Clorhydric Acid 2	2	0.000509421	0.981
Lithium Battery 1	1.3	0.000010684	0.139
Lithium Battery 2	1.3	0.000041618	0.541
Litthium Battery 3	1.3	0.000033723	0.438

Table 3 indicates confirmation of the relationship between the parameters and the RC circuit.

4) DISCUSSION

The results obtained show that the initial pulses in the voltage curves have a behavior that can be described by Green's functions. This reinforces the hypothesis that these functions are useful tools for modelling discharge processes in electrochemical cells and lithium-ion batteries of smartphones. This approach allows the interaction between certain energy storage device characteristics and their behavior to be described mathematically.

5) CONCLUSION

Despite the precision achieved in modelling initial pulses, the study is limited by the controlled conditions under which measurements were made. It would be important to explore whether these characteristics persist in cells subjected to variable environmental conditions or prolonged load-discharge cycles.

The electrochemical model of equivalent RC circuit used allows a comparison of the behavior of lithium-ion batteries in their process of ignition and discharge, adjusting to the mathematical implementation described. This finding is important for its optimization in the management to estimate the state of charge and other possible applications of energy storage.

The modelling of initial impulses as Green's functions opens up the possibility of predicting and including computational mathematical models in the overload effects in electrical installations with devices that are simultaneously switched on. This has significant practical applications in electrical systems engineering, especially in contexts where multiple device synch mic ignition can generate harmful current peaks.

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8) Data Availability: The data that support the findings of this study are available from [José Gabriel Escobedo Gamboa, https://github.com/agsgabo/Article-Dataset]

9) Conflict of interest: The authors declare that there is no conflict of interest.

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11) Miscellaneous:

Appendix A

$$q(t) = \frac{1}{R}\int_0^t V(\tau)\, e^{-(t-\tau)/RC}\, d\tau$$

The voltage across the resistor, V_{Biopac}, is found by Ohm's Law:

$$V_{Biopac}(t) = R\frac{dq}{dt} = \frac{d}{dt}\int_0^t V(\tau)\, e^{-(t-\tau)/RC}\, d\tau$$

Using the Leibniz method of integration, this gives

$$V_{Biopac}(t) = V(\tau) - \frac{1}{RC}\int_0^t V(\tau)\, e^{-(t-\tau)/RC}\, d\tau$$

Integrating by parts, the term $\int_0^t V(\tau)\, e^{-(t-\tau)/RC}\, d\tau$, we obtain [1]

$$\int_0^t V(\tau)\, e^{-(t-\tau)/RC}\, d\tau = RC\left[V(t) - V(0)e^{-t/RC}\right] - \int_0^t \frac{dV(\tau)}{d\tau}\, e^{-(t-\tau)/RC}\, d\tau$$

Then

$$V_{Biopac}(t) = V(0)e^{-\frac{t}{RC}} + \int_0^t \frac{dV(\tau)}{d\tau}\, e^{-(t-\tau)/RC}\, d\tau$$