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ANALYSIS OF THE MATHEMATICAL METHODS HARALICK, GABOR AND WAVELET APPLIED TO THE RECOGNITION OF SOIL TEXTURES

JHON FILIPS RENGIFO AREVALO[1], HENRY FABIAN AGRAMONTE MONGE[2, *], NEICER CAMPOS VASQUEZ[3], RUBEN KEVIN MANTURANO CHIPANA[4]
[1,2,3,4] UNIVERSIDAD PRIVADA DEL NORTE, FACULTAD DE INGENIERÍA, LIMA, PERÚ
[3,4] GRUPO DE INVESTIGACIÓN DESARROLLO E INNOVACIÓN UPN -IDIUPN
CORRESPONDENCE: E-MAIL: neicer.campos@upn.edu.pe

ABSTRACT: Carrying out a mathematical model to determine the type of soil is extremely crucial since it allows us to optimize the effectiveness, precision, and times in the development of recognitions through patterns evaluated in spectral images. The Gabor, Haralick and Wavelet mathematical methods were analyzed, explaining their procedure, complexity, and application for the recognition of soil textures. In this sense, what is sought is to measure the characteristics provided by each model and determine the optimal one. Consequently, after analyzing the Haralick method, it is the most objective because it measures very well how the combinations of pixels are within the image based on the Co-occurrence matrix, including its 14 characteristics, showing an efficiency to determine textures in a spectral image.
Keyword: Gabor***,*** *Haralick****,****Wavelet****,*** *Spectral images, Soli textures*

1. INTRODUCTION

With technological advancement in recent years, new techniques have been implemented in many areas of study, specifically in soil mechanics, so the study of the mechanical properties of soil is more precisely known. The use of mathematical methods in engineering is shown, as it is in the contribution applied to the recognition of soil textures through spectral images around the world.
According to Gabriela Maza, the hyperspectral image provides abundant information from which fundamental characteristics for the analysis of the sample are extracted. This aspect (Maza, 2018) contributes to soil mechanics because it allows to classify and recognize the textures of soils through the application of spectral images.
A clear example of this contribution was made in 2021 in Colombia, the researcher Jonás León determined the spectral images to know what the soil is composed of, the level of minerals that this portion of the study sample has and to know the depth of the strata that the soil will have; In addition, the amount of salts that can be obtained, if the soil is suitable for both agricultural and construction use (Román & Vargas, 2013).
In Peru since 2016, PerúSAT-1 allows the entire population to have access to its satellite images of high spatial resolution to develop research and contribute to the scientific development of engineering (Cologne, 2021). Therefore, it will be possible to investigate the characteristics by means of spectral images by means of software that encompasses mathematical methods to facilitate the process obtaining good results. Therefore, the mathematical methods that will be part of the research are mentioned.
The analysis begins with the Gabor mathematical method, which is a sinusoidal function modulated by a Gaussian envelope. A 3D spectral-spatial Gabor filter is defined in the radiation domain by:

$$\mathrm{g}(\mathrm{x},\mathrm{y},\lambda) = \mathrm{a}(\mathrm{x},\mathrm{y},\lambda)\mathrm{c}(\mathrm{x},\mathrm{y},\lambda) \qquad (1)$$

I should mention that x and y are spatial variables; while λ is the spectral variable e, where the Gaussian component is expressed as follows:

$$a(x,y,\lambda) = \frac{1}{(2\pi)^{3/2}\sigma_x\sigma_y\sigma_\lambda} e^{-0.5\left(\frac{x^2}{\sigma_x{}^2}+\frac{y^2}{\sigma_y{}^2}+\frac{\lambda^2}{\sigma_\lambda{}^2}\right)} \qquad (2)$$

The variables σ x, σ y, σλ describe the size of the Gaussian envelope and define the scale along the spatial and spectral axes. On the other hand, the sinusoidal component is expressed as follows:

$$c(x,y,\lambda) = \cos\left(2\pi\left(F_x x + F_y y + F_z z\right)\right) \qquad (3)$$

The variables F x, F y, Fλ represent the frequency of the sinusoidal components and allow the center frequency of the filter in the 3D frequency domain (Tien, Subhadip, & Glenn, 2008)
This mathematical method examines us of the image is its scale (Gaussian part) and its rotation (sinusoidal part), where we can have "n" scales and "m" rotations, which we take to a matrix nxm where we will have nm characters and additionally the maximum, minimum and average of the charactersss.

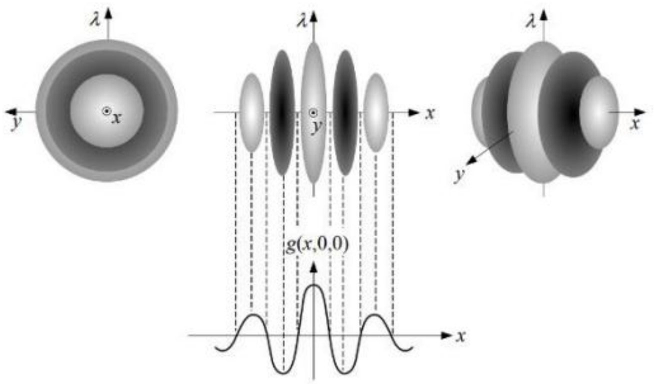


Fig. 1. Gabor 3D filter seen from three different angles. Taken from (Tien, Subhadip, & Glenn, 2008)

The Haralick method is involved for extracting information about textures in an image, this second-order statistical method consists of two steps. The first step is related to the calculation of the GLCM (Gray Level Co-occurrence Matrix) where it is sought to find the number of pixels that are in a correspondence rule; while the second step, the Haralick coefficients are calculated for the characteristics of the image. (Siéler, Tanougast, & Bouridane, 2010).

The following characteristics based on the Co-occurrence matrix are displayed, such as:
Angular second moment (ASM), implies that, the higher ASM, the greater uniformity, and this means that there will be less variation in gray levels.

$$f_1 = \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} [p(i,j)]^2 \qquad (4)$$

Correlation, indicates the linear dependence of the shades of gray in the image.

$$f_2 = \frac{1}{\sigma_x \sigma_y} \sum_{i=1}^{N_x} \sum_{j=1}^{N_x} \left[ij.p(i,j) - \mu_x \mu_y\right]^2 \qquad (5)$$

Maximal correlation coefficient shows how its name itself mentions it, because it is responsible for determining the maximum correlation coefficient in the image.

$$f_3 = \sqrt{\lambda_2} \qquad (6)$$

Features based on the Co-occurrence matrix. Taken from (Haralick, 1979)
The other method that uses mathematical algorithms is the Wavelet Transform, which is an ideal candidate for feature extraction in contexts of classification and process modeling that allows it to be expressed more concisely for its excellent properties for data understanding (Azor, 2013). Of the entire Wavelet family, we will analyze its discrete function expressed in the following equation:

$$DWT_{j,k} = 2^{\frac{j}{2}} \int_{-\infty}^{\infty} x(t) \psi(2^j t - k) dt \qquad (7)$$

Where X(t) is the discrete signal, ψ is the wavelet function, j and k are integer values.

In this sense, the research question is : What is the mathematical method of those mentioned is the most efficient to determine soil textures through spectral images? The research will address and analyze methods that are applied in algorithms for soil characterization.

Therefore, the main objective is to determine which of the three proposed mathematical methods is optimal for application in the recognition of soil textures using spectral imaging.

2. METHODS

In this scientific article, a systematic review is used in a theoretical way. According to J. Pardal and B. Pardal, " The review article offers the researcher and the reader clarifying information on a specific topic; moreover, to be specified in such a way that it must be able to be reproduced by any researcher". (Pardal & Pardal, 2020). For the same reason, various information from reliable sources indexed in databases and academic repositories such as ScienceDirect, EBSCO, ProQuest, Access Engineering, SCOPUS and Scielo will be analyzed.
To carry out the systematic review with respect to databases, it is important to be clear about the interception of keywords such as Wavelet, Haralick, Gabor, soils texture, spectral images. They contributed with the meeting of scientific articles related to the research topic. Also, the information is limited when the keywords in Spanish are placed in the search engine, so it was decided to search in English.
It is directly related to the mathematical methods applied to the determination of soil textures by spectral imaging; Also, the clarity in identifying the variables of the article will contribute to the understanding of the sources in the databases to easily address the research topic.
The selection criteria are as follows:
The search for information is obtained from scientific articles in databases and academic repositories with relevant information on the determination of soil textures applying the mathematical methods Gabor, Haralick and Wavelet. They must be recent articles from the last 15 years and well referenced. In addition, they have to keep a relationship to the career of Civil Engineering either in the specialization of soil mechanics or Geotechnics. The investigations are collected of the first order described by the mathematical methods carried out by the authors themselves as in the case of Haralick.
The criteria for exclusion are as follows:
The search for information from monographs and pages not indexed in databases or academic repositories. Scientific articles that do not address or explain the procedure of mathematical methods applied to determine soil textures by spectral imaging. Evidence or information from second source, non-empirical research. Research or scientific articles over 20 years except research conducted by the author of the mathematical model such as Haralick.

Table 1 presents a statistical table of search of databases and academic repositories by year, obtaining 19 investigations in the last 5 years.

Table 1. Search statistics by year and repository. Taken from: Own elaboration.

Motor de búsqueda / año	2010	2013	2017	2018	2019	2020	2021	2022
ScienceDirect	2	0	1	2	0	0	1	1
Access Engineering	0	1	1	1	1	1	1	0
EBSCO	0	2	0	0	0	1	1	2
SCOPUS	1	0	0	0	0	0	0	0
Scielo	0	0	0	0	0	1	0	0
ProQuest	0	0	0	0	0	0	0	1
Repositorio Institucional Pirhua	0	0	0	1	0	0	0	0
Revista de la Universidad de Mendoza	0	1	0	0	0	0	0	0
Revista UD y la Geomática	0	0	0	0	0	0	1	0
UNMSM	0	0	0	0	0	0	1	0
Total	3	4	2	4	1	3	5	4

Table 2 expresses as a percentage the number of articles found in the databases by means of a pie chart; ScienceDirect being the most used to find research related to mathematical methods to determine soil textures through spectral images.

Table 2. Pie chart expressed as a percentage of the number of items. Taken from: Own elaboration.

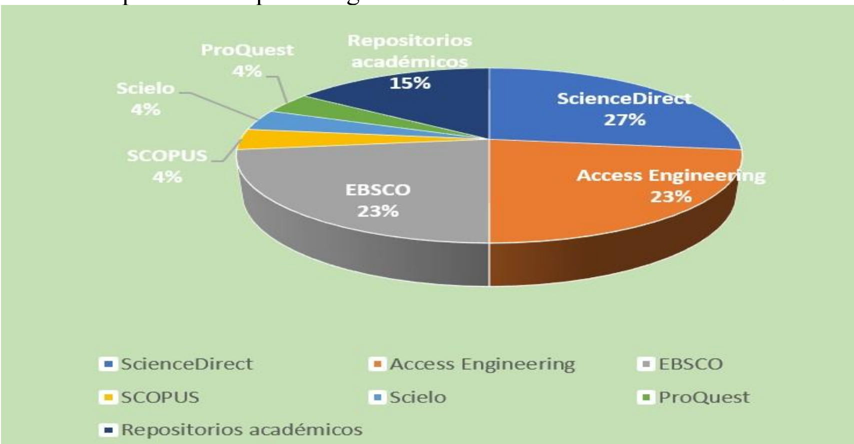


Table 3 shows the search equations for the collection of articles using the combinations of the keywords in the different databases; from which more information from the Haralick mathematical model related to soil textures was found.

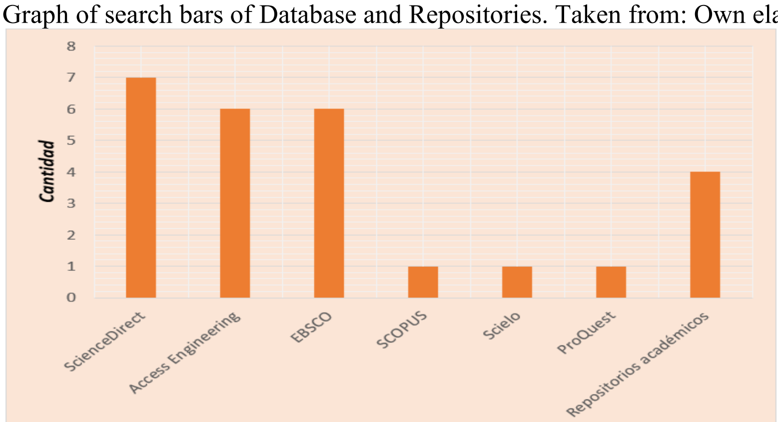
Table 3. Research process. Taken from: Own elaboration

Ecuaciones de búsqueda	*N° artículos*
"Haralick" and "textures" and "soil"	8
"Gabor" and "textures" and "soil"	3
"Wavelet" and "textures" and "soil"	5
"Textures" and "soil"	4
"images" and "spectral" and "soil"	6

3. RESULTS AND DISCUSSION

The analysis of the results obtained in the search of database and academic repositories by year on the analysis of the mathematical methods Haralick, Gabor and Wavelet applied to recognition of soil textures through spectral images is presented, giving a total of 26 scientific articles who are part of the results based on first sources by research of the authors themselves used in mathematical methods, which is disclosed in the following bar chart.

Table 4. Graph of search bars of Database and Repositories. Taken from: Own elaboration.



It is shown that the largest number of articles were found in the ScienceDirect database; however, Access Engineering and EBSCO are the next bases where more articles related to the research topic were found.
We proceed to carry out the results obtained from the articles related to the mathematical models Haralick, Wavelet and Gabor, respectively. Specifying its complexity, its application and how efficient it is for the determination of textures in soils.
The Haralick mathematical method measures very well what the combinations of pixels are like within the image and this leads to coding database in statistics. Mainly, it is based on the Co-occurrence matrices that are responsible for measuring the distribution that two pixels occur simultaneously.
To perform the procedure, a pair of pixels must be analyzed, the first one will be in one position and the second, in the position that the vector B indicates to me. In short, a pixel is analyzed with the one below and that would indicate the direction of the matrix P10, which indicates that 1 should be lowered and 0, means that it does not move to the right or to the left. Therefore, the first pixel is the one that is analyzed with the pair indicated by the vector B.
An example is made with the theory mentioned above. For this practical exercise, an image of a sample with 16 pixels in 3 gray scales (0,1,2) was evaluated, from which it is requested to determine the Co-occurrence matrix P10.

$$I = \begin{bmatrix} 1 & 0 & 2 & 1 \\ 0 & 2 & 1 & 0 \\ 1 & 0 & 0 & 1 \\ 0 & 2 & 0 & 2 \end{bmatrix}$$

$$P_{10} = \frac{1}{12} \begin{array}{c} \\ 0 \\ 1 \\ 2 \end{array} \begin{array}{c} \begin{array}{ccc} 0 & 1 & 2 \end{array} \\ \begin{bmatrix} 1 & 2 & 2 \\ 4 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \end{array}$$

v

Fig.2. Co-occurrence matrix with a 16-pixel image in 3 grayscales. Taken from: own elaboration.
Start by determining how many pixels can be found with respect to the address P10; therefore, there will only be 12 pixels because the last row could no longer apply to this criterion. Then proceed to place the value of the first pixel on the left side vertically and the value of the second pixel at the top horizontally to place the amount that is with the shape of the pixel pairs. To check if it is well resolved, summation must be applied in the matrix that must give the possible number of pairs in the direction that is being calculated and compare it with the normalization that is the one that indicates how many pixels there should be.
But as we see this is a simple, short and practical example; Thus, the following uncertainty appears: What if the diagonal of the co-occurrence matrix is large? This means that the diagonal of that matrix in which the pixels are of the same value will be high, and the direction will not undergo major significant changes. Otherwise, with a diagonal that is small, it will present significant changes.
The Haralick method defines a number of "d" pixels and will show how the distance of the first pixel from the second pixel should be calculated whether the diagonal of the Co-occurrence matrix is large or small. So, you have an image; we proceed to define a number of pixels and calculate the four co-occurrence matrices and each of them takes the 14 elements for the first, second, third and fourth obtaining an average of them and see from those four vectors what is the maximum, minimum and the remains giving a total of 28 elements in which Haralick proposes to determine the textures of an image.

The example of an area in which desert and mountainous parts are observed shown in figure 3 is presented, which will be analyzed by one of the Haralick elements, using characteristic 7 which is the Variance of the sum observed in figure 4. Then, a threshold is made as shown in figure 5 and the final product is obtained where the desert area expressed in figure 6 is detected.

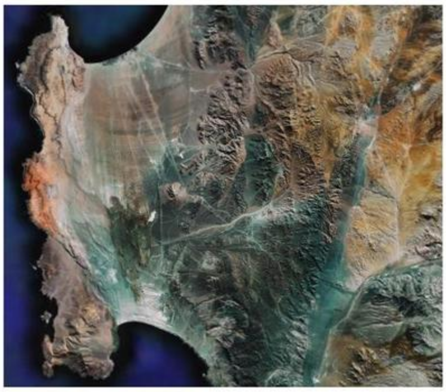


Fig.3. Detection of mountainous and desert areas. Taken from (Patiño, Mery, & Botero, 2012)

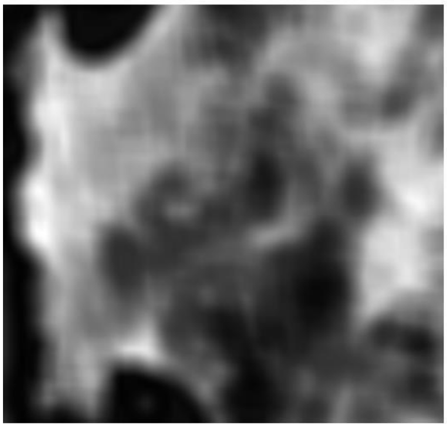


Fig.4. Characteristic 7 of Haralick's method. Variance of the sum. Taken from (Patiño, Mery, & Botero, 2012)

Fig.5. Thresholding of the area. Taken from (Patiño, Mery, & Botero, 2012)

Fig.6. Detection of the desert area. Taken from (Patiño, Mery, & Botero, 2012).

We proceed to analyze the mathematical method of the two-dimensional discrete Wavelet transform in the characteristics of a spectral image applied to soil textures. This model can be decomposed as reconstructing the image, in addition to providing the resolution reduction included in the decomposition process and taking into account that it can also reduce the spatial resolution. It is surprising because based on this, the decomposition and reconstruction of the discrete Wavelet Transform can increase the context information where the selection operation will be performed as a grouping through sampling reduction.

The following figure shows the process of decomposition of the sample. These go through a low and high frequency filtering of which we will have four results that are the decomposed images that have the image information. Now, the inverse of the decomposition would come to do the reconstruction of the image and this, is obtained by means of the inverse transformation along the column and posterior, along the row.

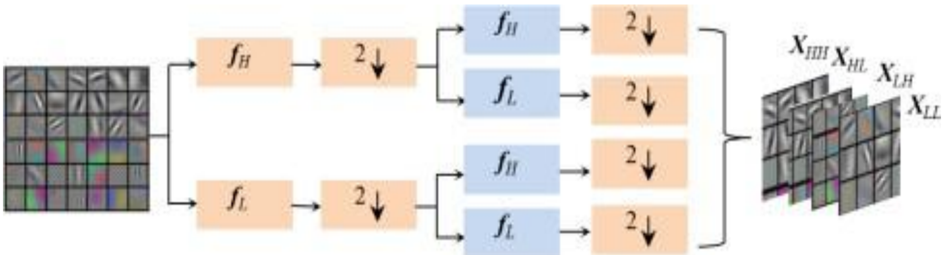


Fig.7. Illustration of the decomposition of the two-dimensional discrete Wavelet Transform image. Taken from (Congcong, Wenbin, Deqin, Xiaoding, & Zhi, 2021).

Let's see a practical case, if you want to have a good transformation you must start by selecting a correct wavelet base and the specifications of what you want to obtain.

The Wavelet base has five fundamental properties which are the moment of leakage, compactly supported form, orthogonality, symmetry and regularity. Starting with the vanishing moment which is the Wavelet order, the greater the disappearance moment, the more concentrated the energy after the Wavelet transformation. Then there is orthogonality, which ensures that there is no redundancy in the decomposition of the signal. Symmetry mentions over a linear phase which prevents phase distortion during image reconstruction. The regularity reflects the smoothness of the Wavelet Transform.

Finally, the Gabor method is analyzed. This consists of two parts, one Gaussian and one sinusoidal, the first will give me the width of the bell while the second the frequency, thus proposing a family of functions that when they are already defined will result in rotation and scale.

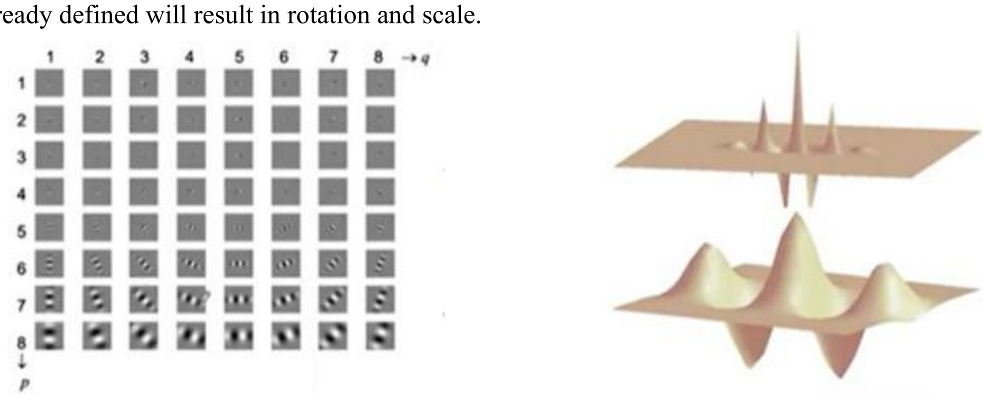


Fig. 8. Rotation scale matrix. Taken from (Rajadell, Garcia-Sevilla, & Pla., 2013).

As you can see in the previous image gabor works with a family of kernels where its rotation and scale varies, when expanding each kernel will give us a series of numbers where if it is a high number it means that there are patterns of that type

In a qxp matrix I will have "nm" invariant characteristics and also 3 additional ones that are variants, an example of how gabor works is by making a classifier that recognizes different textures.

In this example we will take a bank of 9 images and we want to make a classifier of 11 classes. It is taken to gabor and we will obtain a matrix of 999 images x 67 elements of gabor, as well as a vector of 999 kinds of images, we do the separation through training / testing.

The training will consist of the first 8 images per class, which in turn will be subdivided into two parts the Xtrain (matrix of 888x67) and the Ytrain (vector of 888 elements), while the Testing will consist of the characteristics of the last image of the class, the Xtest (matrix of 111x67) and the Ytest (vector of 111 elements) will be part of the Testing.

For the classifier design, we select the xtrain and the ytrain is taken to a train where a trained classifier will get the result. Then for the prediction the test set, this is combined with the xtest will throw us a ypred. And finally, for the performance evaluation the ypred is used together with the ytest having as a final result an accuracy and a confusion matrix.

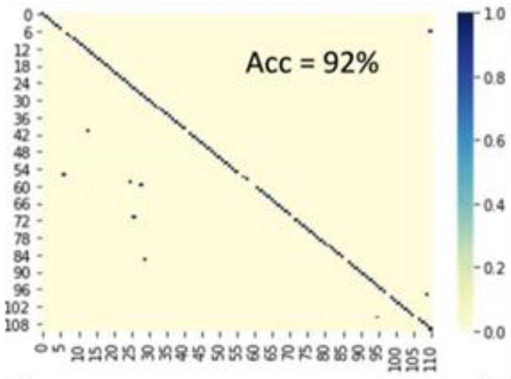
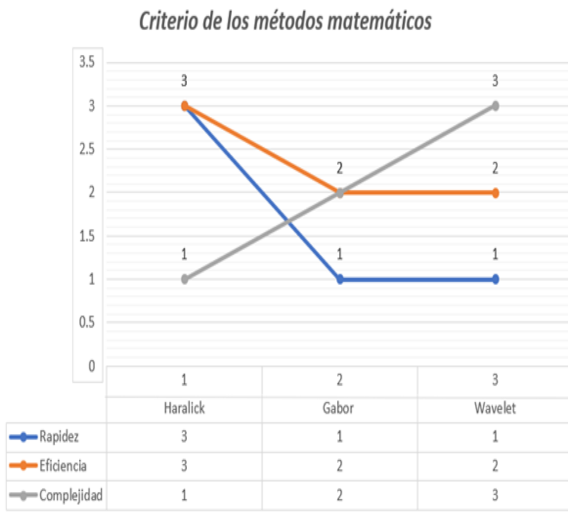


Fig.9. Confusion matrix. Taken from (Rajadell, Garcia-Sevilla, & Pla., 2013)

Accuracy represents the percentage of accuracy that can be in textures. In this image presented it is observed that in texture number 30 generates a confusion with texture number 84.

Table 5 of criteria of the mathematical methods that will be evaluated according to the types of measurement scales such as high, medium and low represented with numbers 3, 2 and 1, respectively, is presented. Which will help us analyze according to three variables, these are through its complexity, its efficiency and the speed of obtaining data. Favorable data is provided for the Haralick method who takes the greatest evaluation of the criteria; Therefore, it satisfies with the functionalities to determine the textures of the soils by means of spectral images.

Table 5. Criteria of mathematical methods. Taken from: Own elaboration.



4. CONCLUSION

The mathematical method Haralick, for the study turns out to be the optimal and who is developed based on the matrix of Co-occurrence where 14 characteristics are evaluated showing an efficiency to determine textures in a spectral image; of which it is notorious, its workability for the same reason that programs are used that facilitate its development and process in the characterization of textures based on the pixels of the images obtained for the analysis of a sample.

On the other hand, the Wavelet method was analyzed, which provides an information-based neural network in which the image is decomposed and reconstructed; however, the problem is in its complexity of analysis because it is tedious to work with respect to the classification of textures in soils and the abundant information that is obtained causing deviations from the main objective that is to determine the textures.

Likewise, with the same criterion the complexity of the Gabor method was observed, who can be applied in our objective to recognize the textures of the soils that works with the confusion matrix to recognize the similarity

resulting in the study soil; however, several softwares are required along with a bank of textures initially to be applied to the determination of textures, presenting difficulty in the elaboration of its procedure.
To evaluate the complexity of the three methods mentioned, it is based on the accuracy of the data, with Haralick being the one with the highest percentage obtaining 90%; while Gabor and Wavelet, 83% and 78%, respectively. Thus, the criteria for this research of the scientific article is chosen, using the Haralick mathematical method for its workability and above all meet the expectations of the main objective in determining the textures of soils through spectral images

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